Summary Ch 6 Interest Rate Shocks

Financial and interest rate shocks are an important source of fluctuations in the EMEs. Until now, we assumed they were all constant (chapters 1-3), or that they grew with the level of debt (debt elastic rates of models 4-5, EDEIR).

Now we identify these shocks empirically and include the structural empirical equations for these shocks in a DSGE model.

Empirically, we see multiple interest rates instead of a single global one due to the presence of **country specific default risk** that leads to country-wise premia. The most common measure of country spreads is the J.P. Morgan's EMBI+ index.

Here we follow Neumeyer and Perri (JME, 2005) and Uribe and Yue (JIE, 2006).

Identification of Interest Rates Shocks

Empirically, we see a negative correlation between the output and interest rates. However, we can't tell which variable drives the other. To understand how the interest rates drives the output we need to identify interest rates shocks.

There are two approaches: 1. Empirical: Estimate SVAR by imposing identifying assumptions and check whether the resulting model describes the data well. 2. Estimate a DSGE model with a well defined interest rate exogenous shock.

Empirical Approach

Follows Uribe and Yue (2006). We set a SVAR:

$$A \begin{bmatrix} \hat{y}_t \\ \hat{\imath}_t \\ tby_t \\ \hat{R}^{us}_t \\ \hat{R}_t \end{bmatrix} = B \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\imath}_{t-1} \\ tby_{t-1} \\ \hat{R}^{us}_{t-1} \\ \hat{R}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon^y_t \\ \epsilon^i_t \\ \epsilon^{tby}_t \\ \epsilon^{rus}_t \\ \epsilon^r_t \end{bmatrix}$$

$\hat{y}$: Output
$\hat{\imath}$: Investment
tby: trade balance to GDP
R^{us}: US int. rate
R : Country specific rate

$\hat{x}$: log-deviation from linear trend.

<u>Idenfitication Assumptions:</u>

<u>A is lower triangular</u>; financial variables are affected by all variables contemporaneously, real variables are affected by financial variables with a lag.

<u>Exogenous Global Rates</u>: Interest rates in the US are not affected by rates at EMEs. $\hat{R}^{us}_t$ Follows a univariate process.

w/ $A(4,j) = B(4,j) = 0$ for $j \neq 4$

Implications: (lower triangular A with A(4,j)=0)

ϵ^{rus}_t : Interpreted as **US interest rate shock**

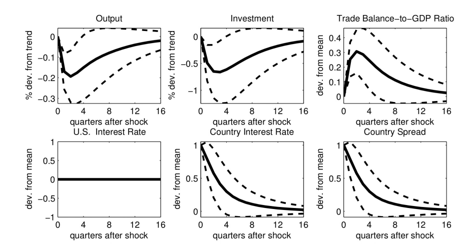
ϵ^{r}_t : Interpreted as **country interest rate shock** or credit spread shock.

Credit spread shock: the spread can be added as a variable, it will be affected by the same shocks affecting the rate, minus those affecting the US rate. The structural error term of the spread will be identical to that of the interest rate given the interest rate of US would show up as a regressor in the equation for the spread.

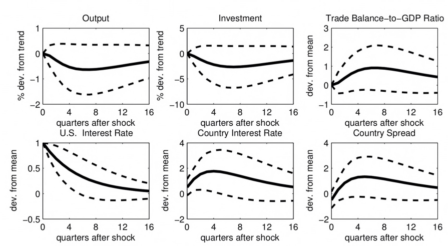
Note on identification: the case of contemporaneous financial effects is explored. In that case the model generates a positive correlation between interest rates and output which is hard to justify.

Note on estimation: this will be a panel SVAR whose estimation is done equation by equation using OLS fixed effects panel. Structural part (A matrix) dictates what contemporaneous terms are included in the RHS of each of the five equations.

Impulse Response To A Country-Spread Shock, ϵ^r_t



Impulse Response To A U.S. Interest-Rate Shock, ϵ^{rus}_t



Both interest rates shocks generate contractions (and improvements in trade balance)

US interest rate shocks leads to a large and delayed overshooting of country spreads

Output shock (IRF not shown here) drives down the country spread. It is hard to say why in this case as the nature of that shock in this SVAR is fuzzy (the identified ones are the interest rates ones)

FEVD: Forecast Error Variance Decompositions

let $x_t = [\hat{y}_t, \hat{\imath}_t, tby_t, \hat{R}^{us}_t, \hat{R}_t]'$, the SVAR can be written as $A x_{t+h} = B x_{t+h-1} + \epsilon_{t+h}$.

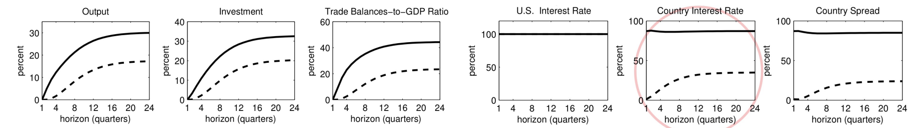
SVAR MA(∞) representation: $x_{t+h} = \sum_{j=0}^{\infty} C_j \epsilon_{t+h-j}$ w/ $C_j = (A^{-1}B)^j A^{-1}$; The forecast of x_{t+h} is: $E_t x_{t+h} = \sum_{j=h}^{\infty} C_j \epsilon_{t+h-j}$

Then the forecast error is: $FE^h_t = \sum_{j=0}^{h-1} C_j \epsilon_{t+h-j}$; The Forecast Error Variance at horizon h is: $FEV^h = \sum_{j=0}^{h-1} C_j \Sigma_\epsilon C_j'$ w/ $\Sigma_\epsilon = E[\epsilon_t \epsilon_t']$

The forecast error attributable to a shock i is: $FEV^{h,i} = \sum_{j=0}^{h-1} (C_j \Lambda_i) \Sigma_\epsilon (C_j \Lambda_i)'$, w/ Λ_i : Conformable square matrix w/ zeros in all values but diagonal element (i,i) which equals one

Then the share of forecast error variance is: $SFEV^{h,i}_k = \frac{FEV^{h,i}_{kk}}{FEV^h_{kk}}$, w/ kk denoting the k-th diagonal element

Estimated Forecast-Error Variance Decomposition



Result: Country interest rate is driven mostly by country spread shocks

Interest rate shocks (local and US) explain between 30-44% of real variables and about 60% of country spreads.

Theoretical approach: DSGE analysis

Strategy: 1) Build DSGE (set model and estimate/calibrate parameters), 2) Feed model with estimated interest rate processes for R-us and R (last two equations of SVAR), 3) Compare impulse responses of SVAR and DSGE

Note on 2): This is called a limited information method. Normally we either assumed the interest rates were constant, or provided an equation for them to complete the model (EDEIR), other approaches use policy rules. Here, we will provide equations for these rates too but based entirely on the SVAR result.

Special Features and Departures from a standard RBC

1. Innovations in R-us and R are assumed to have lagged effects (many choice variables will be t+1 variables)
2. Preferences display <u>external consumption habits</u> (more consumption smoothing)
3. <u>Gestation lag</u> in capital accumulation and adjustment costs (increase investment smoothing and persistence)
4. <u>Working capital constraint</u> (induce supply side effects of interest rate shocks - cost of labor increases with int. rate hikes & lowers labor demand - generates more realistic responses of output to interest rate shocks)

Model

Technology: $y_t = F(k_t, h_t)$ (6.2)

Problem of Firms

Choose labor, capital to Max $F(k_t,h_t) - u_t k_t - w_t h_t \Big[1 + \underbrace{\frac{\eta(R^d_t - 1)}{R^d_t}}_{\text{Working Capital Constraint (binding)}}\Big]$ w/ u_t: Rental Rate of Capital, w_t: Wage, R^d_t: Gross interest rate

The resulting optimal conditions dictate the factor demands: $F_h(k_t,h_t) = w_t \left[1 + \eta\left(\frac{R^d_t - 1}{R^d_t}\right)\right]$ (6.3) and $F_k(k_t, h_t) = u_t$ (6.4)

Capital Accumulation Dynamics

Gestation lags: Gross Investment i_t comes from the investment projects of four periods: s_{it}: Number of projects started in t-i for $i=0,1,2,3$

$i_t = \frac{1}{4}\sum_{i=0}^{3} s_{it}$ (6.5) w/ $s_{i+1,t+1} = s_{it}$ (6.6) $i=0,1,2$ (e.g. for t=1 $s_{1,2} = s_{0,1}$; $s_{2,2} = s_{1,1}$; $s_{3,2} = s_{2,1}$)

Stock of Capital: $k_{t+1} = (1-\delta)k_t + k_t \Phi\left(\frac{s_{3t}}{k_t}\right)$ (6.7)

Households: Choose $c_{t+1}, h_{t+1}, d_t, s_{0,t+1}, s_{1,t+1}, s_{2,t+1}, s_{3,t+1}, k_{t+1}$ (Information timing effect assumption: HH choose consumption, labor supply, and investment one period in advance)
(also: external consumption habits)

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big\{ U(c_t - \mu\tilde{c}_{t-1}, h_t) + \lambda_t \Big[d_t - R_{t-1}d_{t-1} - \Psi(d_t) + w_t h_t + u_t k_t + \pi_t - \underbrace{\tfrac{1}{4}\sum_{i=0}^{3} s_{it}}_{i_t} - c_t\Big]$$
$$+ \lambda_t q_t \Big[(1-\delta)k_t + k_t \Phi\Big(\frac{s_{3t}}{k_t}\Big) - k_{t+1}\Big] + \lambda_t \sum_{i=0}^{2} v_{it}(s_{it} - s_{i+1,t+1}) \Big\}$$

FOCs:

$[c_{t+1}]$: $\mathbb{E}_t \lambda_{t+1} = U_c(c_{t+1} - \mu\tilde{c}_t, h_{t+1})$

$[h_{t+1}]$: $\mathbb{E}_t[w_{t+1}\lambda_{t+1}] = -U_h(c_{t+1} - \mu\tilde{c}_t, h_{t+1})$

$[d_t]$: $\lambda_t[1 - \Psi'(d_t)] = \beta R_t \mathbb{E}_t \lambda_{t+1}$

$[k_{t+1}]$: $\lambda_t q_t = \beta \mathbb{E}_t \Big\{ \lambda_{t+1} q_{t+1} \Big[1 - \delta + \Phi\Big(\frac{s_{3t+1}}{k_{t+1}}\Big) - \frac{s_{3t+1}}{k_{t+1}} \Phi'\Big(\frac{s_{3t+1}}{k_{t+1}}\Big)\Big] + \lambda_{t+1} u_{t+1} \Big\}$

$[s_{0,t+1}]$: $\mathbb{E}_t \lambda_{t+1} v_{0t+1} = \frac{1}{4}\mathbb{E}_t \lambda_{t+1}$

$[s_{1,t+1}]$: $\beta \mathbb{E}_t \lambda_{t+1} v_{1,t+1} = \frac{\beta}{4}\mathbb{E}_t \lambda_{t+1} + \lambda_t v_{0t}$

$[s_{2,t+1}]$: $\beta \mathbb{E}_t \lambda_{t+1} v_{2,t+1} = \frac{\beta}{4}\mathbb{E}_t \lambda_{t+1} + \lambda_t v_{1t}$

$[s_{3,t+1}]$: $\beta \mathbb{E}_t \Big[\lambda_{t+1} q_{t+1} \Phi'\Big(\frac{s_{3t+1}}{k_{t+1}}\Big)\Big] = \frac{\beta}{4}\mathbb{E}_t \lambda_{t+1} + \lambda_t v_{2t}$

Pricing of investment Projects at different Stages of completion

Empirically Identified Processes for Interest Rates

$$\hat{R}_t = 0.635\,\hat{R}_{t-1} + 0.501\,\hat{R}_t^{us} + 0.355\,\hat{R}_{t-1}^{us} - 0.791\,\hat{y}_t + 0.617\,\hat{y}_{t-1} + 0.114\,\hat{i}_t - 0.122\,\hat{i}_{t-1} + 0.288\,tby_t - 0.190\,tby_{t-1} + \epsilon_t^r$$

w/ $tby_t = \dfrac{y_t - c_t - i_t - \psi(d_t)}{y_t}$

$$\hat{R}_t^{us} = 0.830\,\hat{R}_{t-1}^{us} + \epsilon_t^{us}$$

Note: we need to include an equation for the interest rates. Other models normally include a policy rule, assume they are constant (as in earlier chapters), or include another functional form like the EDEIR from previous chapters. Alternatively, we can include an estimated equation (limited information method) as done here with the SVAR.

Equilibrium

Due to symmetry $c_t = \tilde{c}_t$

Aggregate Resource Constraint: $d_t = R_{t-1} d_{t-1} + \psi(d_t) + c_t + i_t - y_t$

Definition of Equilibrium: $c_{t+1}, \tilde{c}_t, h_{t+1}, d_t, i_t, k_{t+1}, s_{it+1}\ (i=0,1,2,3), R_t, R_t^d, R_t^{us}, w_t, u_t, y_t, tby_t, \lambda_t, q_t, v_{it}\ (i=0,1,2)$ for $t\geq 0$ Satisfying (6.2) - (6.7), (6.11) - (6.24), given $c_0, y_{-1}, i_{-1}, h_0, k_0, d_{-1}, tby_{-1}, R_{-1}, R_{-1}^{us}, \epsilon_t^{rus}, \epsilon_t^r$

Calibration: $\omega = 1.45,\ \gamma = 2,\ \alpha = 0.32,\ R = \beta^{-1} = 1.0277,\ \delta = 0.025,\ \frac{tb}{y} = 0.02$

Estimation: $\Theta = \{\psi, \phi, \eta, \mu\}$ Estimated by Impulse response matching (limited information approach)

IRF matching: pick Θ to minimize the distance between empirical and theoretical IRFs

$$\min_{\Theta}\ [IR^{emp} - IR^{model}(\Theta)]\ \underbrace{\Sigma^{-1}_{IR^{emp}}}_{\text{Variance of empirical IRFs along the diagonal}}\ [IR^{emp} - IR^{model}]$$

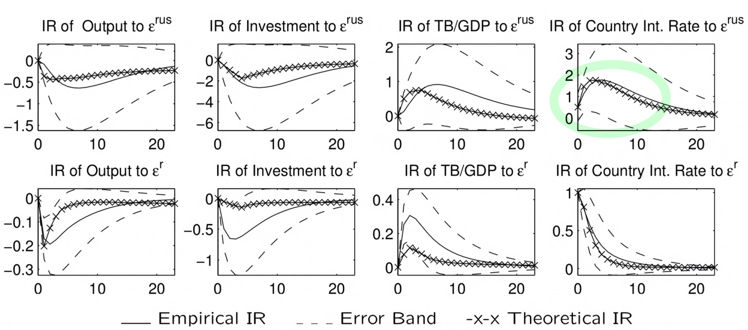
Details: 4 parameters are set to match 185 points

7 degrees of freedom lost: 6 zero valued contemporaneous responses and a 1 valued response of shock on itself

Result: Debt costs are negligible $\psi = 0.00042$

Capital adjustment costs are important $\phi = 72.8$

Comparison of IRFs:



The model successfully replicates 3 features of the SVAR

1. Negative effect of interest rate shocks on y, i, c
2. Positive effect on trade balance
3. Delayed (hump shaped) response of country interest rate to US Rate

Effects of Global Risk Premia Shocks:

Akinci (2013) expands Uribe and Yue (2006) SVAR by including the US Spread. The Spread is approximated as the US Baa Corporate bonds rate minus the 20 year US-T-Bill (Baa bond will have about a 13% risk of default over 20 years)

Identification:

For other variables the same as UY2006.

Now $[\hat{S}_t^{us}, \hat{R}_t^{us}]$ follows a bivariate AR process. Then ϵ_t^{sus} can be interpreted as an innovation to the US risk premium.

$$A \begin{bmatrix} \hat{y}_t \\ \hat{\imath}_t \\ tby_t \\ \hat{R}_t^{us} \\ \hat{S}_t^{us} \\ \hat{R}_t \end{bmatrix} = B(L) \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\imath}_{t-1} \\ tby_{t-1} \\ \hat{R}_{t-1}^{us} \\ \hat{S}_{t-1}^{us} \\ \hat{R}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t^y \\ \epsilon_t^i \\ \epsilon_t^{tby} \\ \epsilon_t^{rus} \\ \epsilon_t^{sus} \\ \epsilon_t^r \end{bmatrix}$$

The results are similar to UY2006:

Now interest rates and Global Premia explain about 42% of the output variance.

Country Spread continues to be an important driver of the EME fluctuations.

The global premia is taking part of the role played by the US interest rate in the simpler UY2006 model.