Summary Ch 4 Open Economy Real Business Cycles Model

Before: w/ physical capital and AR(1) shocks the SOE model can explain the countercyclical trade balance.

Now: Can it also explain the other business cycles properties?

Examples: $\sigma_{tb} > 0$, Serial Corr., $\sigma_i > \sigma_y$

To give a better chance, add to the model: Elastic Labor Supply, Uncertainty in technology shocks, Capital depreciation $\Rightarrow$ the model become the SOE-RBC

Model:

Max $E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t)$ (4.1)

subject to

$$c_t + i_t + \underbrace{\Phi(k_{t+1} - k_t)}^{\text{Adjustment costs}} + (1 + \underbrace{r_{t-1}}_{\text{Int rate no longer constant}})d_{t-1} = y_t + d_t \quad (4.2)$$

$$y_t = A_t F(k_t, h_t) \quad (4.3)$$

$$k_{t+1} = (1 - \delta)k_t + i_t \quad (4.4)$$

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1 + r_s)} \le 0 \quad (4.5)$$

Stationarity: Ct ~ RW
In the previous model consumption is a random walk (not stationary). This is troublesome, the steady state becomes history dependent. The model is valid but after a shock there is no guarantee of convergence to the SS it was approximated around.

Also, the variables stop having well defined moments. For that, we must adjust the model to induce stationarity.
Strategy used here: time varying risk premium in rate of debt (then time varying rate, s.t. Ct is no longer a RW)

drop i_t by substituting, then take FOCs

To make things simpler, here we set a *centralized* version of the model where all decisions are taken by the household (same conditions hold with firms; pag. 77).

Household's Optimality Conditions

$[\lambda_t]$: $$c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t) + (1 + r_{t-1})d_{t-1} = A_t F(k_t, h_t) + d_t \quad (4.6)$$

$[d_t]$: $$\lambda_t = \beta(1 + r_t)E_t\lambda_{t+1} \quad (4.7)$$

$[c_t]$: $$U_c(c_t, h_t) = \lambda_t \quad (4.8)$$

$[h_t]$: $$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \quad (4.9)$$

$[k_{t+1}]$: $$1 + \Phi'(k_{t+1} - k_t) = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \quad (4.10)$$

add $r_t = r^* + p(\tilde{d}_t)$ w/ $\tilde{d}_t = d_t$ to induce stationarity and a unique solution, then drop λ_t:

$\tilde{d}_t$ is taken as given when taking FOCs then replaced by d_t

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = A_t F_h(k_t, h_t) \quad (4.11)$$

$$c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t) + [1 + r^* + p(d_{t-1})]d_{t-1} = A_t F(k_t, h_t) + d_t \quad (4.16)$$

$$U_c(c_t, h_t) = \beta(1 + r^* + p(d_t))E_t U_c(c_{t+1}, h_{t+1}) \quad (4.17)$$

$$1 = \beta E_t \left\{ \frac{U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} \frac{[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]}{1 + \Phi'(k_{t+1} - k_t)} \right\} \quad (4.18)$$

This is a second order difference system (since it has k_{t+2}, k_{t+1}, k_t). To make it a first order difference system add one equation: $k^f_t = k_{t+1}$ ($\Rightarrow k_{t+2} = k^f_{t+1}$) (This is what Dynare always does)

Now we have a 1st order difference system of eqs. in $k^f_t, k_t, c_t, d_{t-1}, y_t$

Solution: $y_t = g(x_t, \sigma)$ $\quad$ y_t: Non predetermined variables $\quad$ (w/ $\sigma = 0$: deterministic)

$x_{t+1} = h(x_t, \sigma) + \sigma \epsilon_{t+1}$ $\quad$ x_t: States: Predetermined & Exogenous variables

Solution will be obtained w/ a 1st order Taylor Exp. approximation around SS of $y_t, x_t, \sigma = 1$:

Solution (Approx.): $\hat{y}_t = g_x \hat{x}_t$

$\hat{x}_{t+1} = h_x \hat{x}_t + \eta \epsilon_{t+1}$

To continue we specify functional forms:

CRRA GHH: $U(c,h) = \dfrac{G(c,h)^{1-\sigma} - 1}{1-\sigma}$ $\quad$ $G(c,h) = c - \dfrac{h^\omega}{\omega}$, $\omega > 1, \sigma > 0$ (Note: w/ GHH the wage won't depend on C_t $\Rightarrow$ No wealth effect in labor decisions)

Debt Elast. Int. Rate: $p(d) = \psi(e^{d-\bar{d}} - 1)$, $\psi > 0$ $\quad$ P/n f/n: $F(k,h) = k^\alpha h^{1-\alpha}$, $\alpha > 0$ $\quad$ Adj. Cost: $\Phi(x) = \frac{\phi}{2}x^2$, $\phi > 0$

$\Rightarrow$ Params: $\omega, \sigma, \alpha, \phi, \bar{d}, \psi$

Calibration:

We need to calibrate $[\omega \;\; \alpha \;\; \delta \;\; r^* \;\; \beta \;\; \sigma \;\; \phi \;\; \rho \;\; \tilde{\eta} \;\; \bar{d} \;\; \psi]$

We use 3 types of strategies (restrictions)

A. Give values to parameters from external sources (not the particular data being explained)
(from Lit: $\sigma = 2$, $\delta = 0.1$, $r^* = 0.04$, $\beta = 1/(1+r^*)$)

B. Target First Moments. Here: labor share = 0.68, tb/y = 0.02

C. Give values to target Second Moments. Here: $\sigma_y, \sigma_h, \sigma_i, \sigma_{Tb/y}, \rho_{\ln y_t, \ln y_{t-1}}$

Some values in B imply a parameter values in a straightforward way (e.g., w/ labor share of 0.68: $\alpha = 0.32$)

For the rest:

Step 1: Let the remaining parameters to define be $\Theta = [\omega \;\; \bar{d} \;\; \phi \;\; \psi \;\; \rho \;\; \tilde{\eta}]$

Guess values for all but 1 of the parameters in Θ (here $\bar{d}$)

Steps 2-4: Solve analytically for the remaining parameter and steady state.

2. Given a guess for ω find h (SS of labor)

   First get k/h from Euler equation: $1 = \beta[\alpha(k/h)^{\alpha-1} + 1 - \delta]$

   plug it into the labor market equation: $h^{\omega-1} = (1-\alpha)(k/h)^\alpha$

   Solve for h; with h solve for k = (k/h)h, and for y via SS of production function: $y = A(k/h)^\alpha h$

3. Let s_{tb} be the average trade balance-to output ratio (that we found in B as 0.02)

   in SS: $s_{tb} = \dfrac{r^*\bar{d}}{y}$ (using the fact that $d = \bar{d}$, or $p(d) = 0$), get: $\bar{d} = s_{tb} y / r^*$

4. Find c from the resource constraint: $c + \delta k + r^* + \psi(e^{d-\bar{d}} - 1)d = y$

Step 5: w/ SS (c, k, h, d) and parameters at hand compute second moments predicted by the model

(Theoretical moments, e.g., from Var matrix) $\quad$ $\chi(\Theta) = [\sigma_y \; \sigma_h \; \sigma_i \; \sigma_{tb/y} \; Corr(\ln y, \ln y_{t-1})]$

Step 6: Compute distance between model's implied moments and targeted moments: $D(\Theta) = [\chi(\Theta) - \chi^*]$

Step 7: Repeat by adjusting guess for Θ until D is small (D < D* -threshold-)

After calibration we compare the model and data

	Canadian Data 1946 to 1985			Canadian Data 1960 to 2011			Model		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,y_t}
y	2.8	0.6	1	3.7	0.9	1	$\boxed{3.1}$	$\boxed{0.6}$	1
c	2.5	0.7	0.6	2.2	0.7	0.6	2.7	0.8	0.8
i	9.8	0.3	0.6	10.3	0.7	0.8	$\boxed{9.0}$	0.1	0.7
h	2.0	0.5	0.8	3.6	0.7	0.8	$\boxed{2.1}$	0.6	1
$\frac{tb}{y}$	1.9	0.7	-0.1	1.7	0.8	0.1	$\boxed{1.8}$	0.5	-0.04
$\frac{ca}{y}$							1.4	0.3	0.05

$\Box \rightarrow$ Calibration targets
$\therefore$ it's normal fit is good, can't be used to test the model
test 1: Model places σ_c well
$\sigma_{tb/y} < \sigma_h < \sigma_c < \sigma_y < \sigma_i$
test 2: tb/y is countercyclical

Problem: model overestimates correlation of h, c, with y

Corr(yt,ht)=1 due to GHH preferences and log-linearized intra-temporal Euler Equation

$\omega \hat{h}_t = \hat{y}_t$

Other Stationarity Inducing Methods:

Complete Markets in Assets: we add state-consumption smoothing (before only time smoothing)
Main change: Budget constraint (and then Euler equations)

$$E_t q_{t,t+1} b_{t+1} = b_t + A_t F(k_t, h_t) - c_t - i_t - \Phi(k_{t+1} - k_t)$$

q_{t,t+1}: SDF

W/ complete markets the future assets keep the expectation operator. It denotes the aggregation of state-contingent (value of) assets, times the probability of the states.

Other stationarity inducing devices (continued)

notice the timing

	In LHS	in RHS
Incomplete Mkts	$(1+r_{t-1})d_{t-1}$ → what you get in t (Proceeds from your investment in $t-1$)	d_t → what you buy/invest in t so that you get $(1+r_t)b_t$ in $t+1$
Complete Mkts	$\mathbb{E}_t \underbrace{q_{t+1}}_{SDF = q_{t,t+1}} b_{t+1}$ → what you buy in t for Future periods (many State contingent assets s.t. in expectation you get b_{t+1})	b_t → what you get in t

Assumption: Frictionless international markets, then UIP holds $r_{t+1} = r^*_{t+1}$

For foreign agents: $[b^*_{t+1}]:\ \lambda^*_t q_{t+1} = \beta \lambda^*_{t+1}$

For domestic agents: $[b_{t+1}]:\ \lambda_t q_{t+1} = \beta \lambda_{t+1}$

$\Big\}\ \lambda_t = \underbrace{\xi \lambda^*_t}_{\text{Constant for SOE}}$

Then we get a Perfect Risk Sharing Condition that holds <u>w/o Expectations</u>

MUCt (or Ct if CRRA) is no longer a RW; now it's constant (and stationary)
(In non SOE environments similar conclusions hold but in terms of consumption differential)

To recap: Incomplete markets: $U_c(C_t,h_t) = E_t U_c(C_{t+1},h_{t+1})$ (w/ $\beta(1+r)=1$)

Complete markets: $U_c(C_t,h_t) = \text{Constant}$

Current account and trade balance in CAM:

$$CA_t = Tb_t + \underbrace{b_t - E_{t-1} q_{t-1} b_t}_{\text{Net invest. income}}$$

$$Tb_t = y_t - C_t - i_t - \phi(k_{t+1}-k_t) \quad (\text{or } S_t = b_t + Tb_t, \text{ w/ } S_t = E_t q_{t+1} b_{t+1})$$

<u>Internal</u> Debt Elastic Interest Rate (IDEIR):

The debt in the premium is a decision variable (or internalized as such)

$$r_t = r + p(d_t) \quad (\text{before: } \tilde{d}_t \text{ (average - exog)})$$

Euler equation changes: $\lambda_t = \beta[1 + r + p(d_t) + \underline{p'(d_t) d_t}]\, \mathbb{E}_t \lambda_{t+1}$

Calibration is the same as EDEIR but SS value of d changes.

External Discount Factor

Induce stationarity by having a time-varying discount factor.

The discount factor will depend on the economy variables.

UMP: $\max E_0 \sum_{t=0}^{\infty} \Theta_t U(C_t, h_t)$

$$\Theta_{t+1} = \beta(\underbrace{\tilde{C}_t, \tilde{h}_t}_{\text{taken as given}})\Theta_t, \quad \Theta_0 = 1$$

(aggregate)

In equilibrium: $C_t = \tilde{C}_t,\ h_t = \tilde{h}_t$

Internal Discount Factor

The DF now becomes a decision variable (as agents now internalize the effects of their choices in the DF)

$$\Theta_{t+1} = \beta(C_t, h_t)\Theta_t, \quad \Theta_0 = 1$$

Θ_{t+1} is now a choice variable

Portfolio Adjustment Cost Model (PAC)

Include adjustment costs of debt in budget constraint:

$$d_t = (1+r_{t-1})d_{t-1} - y_t + c_t + i_t + \phi(k_{t+1}-k_t) + \underline{\frac{\psi_3}{2}(d_t - \bar{d})^2}$$

Effect: Euler equation depends on assets

$$\lambda_t[1 - \psi_3(d_t - \bar{d})] = \beta(1+r_t)\mathbb{E}_t \lambda_{t+1}$$

Note: a time varying interest rate as a stationarity inducing device works in an SOE. However, it does not work in a LOE setup.

Here it prevents having Ct = Et[Ct+1] (int. rate no longer cancels out)

However, in LOE models we still have that Ct^D = Et[Ct+1^D] as the rate cancels out between countries (C^D: consumption differential). Thus, in the LOE we need other stationarity inducing device, for example, adjustment costs on the bonds.

In the LOE what we want in addition (to have stationarity) is for the Euler eq. to depend on the assets (bonds or debt).

Perpetual Youth and OLG approaches

Alternatively, we can consider approaches where the debt has a self-stabilizing mechanism. For example, setups where cohorts of agents are born without debt (or assets) and other die so some assets disappear. With this feature, even if individual variables are random-walks an stationary debt induced stationarity of other variables

Global Solution Method

We can also consider as alternative solutions that do not depend too much on approximations around the steady state. The model is identical to the baseline SOE except we assume:

More impatience is usually assumed in globally solved models as higher order terms now show up and when added imply stationarity (precautionary savings create a well defined debt distribution)

The solution algorithm is Value Function Iteration. It implies setting the model recursively (Bellman equation):

$$V(d,k,A) = \max_{\{d',k'\}} \Big\{ U\Big(AF(k,h) + (1-\delta)k - k' - \phi(k'-k) + d' - (1+r^*)d - \frac{h^\omega}{\omega}\Big) + \beta E[V(d',k',A') \mid A] \Big\}$$

$$\text{s.t. } d' \leq \bar{d} \quad \text{(upper limit of debt, set large)}$$

The state space is discredited with 9 points for lnA from -0..4495 to 0.04495, 70 points for d, and 30 for k.

$\beta(1+r^*)$ set far enough from 1 (0.9922).

With d large (borrowing constraint does not bind):

$d \in [7.45, 9.95] \rightarrow k \in [2.8, 3.8]$

Here the results are not supported by the data (too high tb/y and volatility of investment). Reason: with high debt the intertemporal rate of substitution is too volatile. Then the upper bound on debt is set at 1

$\bar{d} = 1$

Comparison between models

The approaches yield very similar moments and IRF dynamics, implying that most stationarity inducing devices have similar implications.

Only salient difference: CAM model. Due to constant MUC property the model yields a more stable consumption and is unable to generate a countercyclical trade balance (to GDP). In this case, the only variable offsetting the effect of income increases is the investment. It achieves a negative response on impact (IRF) but is not enough to generate a negative correlation over the business cycle (moments).

Extra: Finding Second Moments of the model

Once the solution is obtained, we can get the implied second moments of the model as follows:

Model Solution: $\hat{y}_t = g_x \hat{x}_t$

$$\hat{x}_{t+1} = h_x \hat{x}_t + \eta \epsilon_{t+1}$$

We can get the variance of X: Σ_x

$$\Sigma_x = E\hat{x}_t\hat{x}_t' = E(h_x\hat{x}_{t-1} + \eta\epsilon_t)(h_x\hat{x}_{t-1} + \eta\epsilon_t)'$$

$$= E\{h_x\hat{x}_{t-1}\hat{x}_{t-1}'h_x' + h_x\hat{x}_{t-1}\cancel{\epsilon_t'}\eta' + \eta\cancel{\epsilon_t}\hat{x}_{t-1}'h_x' + \eta\epsilon_t\epsilon_t'\eta'\}$$

$$= h_x E[\hat{x}_{t-1}\hat{x}_{t-1}']h_x' + \eta E(\cancel{\epsilon_t\epsilon_t'})\eta'$$

$$= h_x\Sigma_x h_x' + \Sigma_\epsilon \quad (\text{w/ } \Sigma_\epsilon = \eta\eta')$$

We apply the vec operator: (and $Vec(ABC) = (C'\otimes A)Vec(B)$)

$$vec(\Sigma_x) = vec(h_x\Sigma_x h_x) + vec(\Sigma_\epsilon)$$

$$= \mathcal{F}\, vec(\Sigma_x) + vec(\Sigma_\epsilon)$$

$$\Rightarrow vec(\Sigma_x) = (\mathbb{I} - \mathcal{F})^{-1} vec(\Sigma_\epsilon)$$

$$\Sigma_y = g_x \Sigma_x g_x'$$