Summary Ch 2 — An Open Endowment Economy

Simple enough to be solved analytically.
Households receive an exogenous stochastic endowment (perishable) each period.
HH access an internationally traded bond (to smooth consumption)

Consumption smoothing leads to Eq. Movements in trade balance (current account).
Thus the name of this framework: **Intertemporal Approach to Current Account.**

Model

Household: Solves $\max_{c_t, d_t} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$

s.t. $c_t + (1+r) d_{t-1} = y_t + d_t$ [BC]

$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1+r)^j} \le 0$ [No Ponzi] (binds in Eq.) (TVC)

c_t: Consumption $\quad d_t$: debt (bond)

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{ u(c_t) + \lambda_t [d_t + y_t - (1+r) d_{t-1} - c_t] \}$$

FOC: $[c_t]$: $u'(c_t) = \lambda_t$

$[d_t]$: $\lambda_t = \beta(1+r) E_t \lambda_{t+1}$

Euler Equation: $u'(c_t) = \beta(1+r) E_t u'(c_{t+1})$

Intertemporal Resource Constraint

Write the BC for period t+j, divide by (1+r)^j and take expected values conditional to t:

$$\frac{E_t c_{t+j}}{(1+r)^j} + \frac{E_t d_{t-1+j}}{(1+r)^{j-1}} = \frac{E_t y_{t+j}}{(1+r)^j} + \frac{E_t d_{t+j}}{(1+r)^j}$$, Sum from $j=0$ to $j=J \longrightarrow \sum_{j=0}^{J} \frac{E_t c_{t+j}}{(1+r)^j} + (1+r) d_{t-1} = \sum_{j=0}^{J} \frac{E_t y_{t+j}}{(1+r)^j} + \underbrace{\frac{E_t d_{t+j}}{(1+r)^j}}_{=0 \text{ as } J\to\infty \text{ (TVC)}}$

then: $$(1+r) d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t (y_{t+j} - c_{t+j})}{(1+r)^j} \qquad (2.5)$$

Trade Deficit:

Given single good assumption: $tb_t = y_t - c_t$ (tb: trade balance)

$\Rightarrow$ Can rewrite (2.5): $(1+r) d_{t-1} = \sum_{j=0}^{\infty} \frac{E_t tb_{t+j}}{(1+r)^j}$

If the economy is a net debtor (d_{t-1} > 0) then at least once it must be that tb_{t+j} > 0
=> **An economy cannot run perpetual deficits**

Two Simplifying Assumptions (to find a closed-form solution)

(1) subjective discount rate = market rate $\quad \beta = \frac{1}{1+r}$

(2) Quadratic utility $\quad u(c) = -\frac{1}{2}(c_t - \bar{c})^2 \quad$ (Model becomes Hall (78))

$\Rightarrow$ Euler Equation: $c_t = E_t c_{t+1}$ $\qquad$ Consumption follows a random Walk $\quad E_t c_{t+j} = c_t \quad \forall j > 0$

Replace $E_t c_{t+j} = c_t$ in (2.5) and solve for c_t

$$(1+r) d_{t-1} = \sum_{j=1}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} - c_t \underbrace{\sum_{j=1}^{\infty} \frac{1}{\cancel{(1+r)^j}}}_{\frac{1+r}{r}} \Rightarrow c_t = \overbrace{\frac{r}{1+r} \sum_{j=1}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}}^{y_t^p} - r \cdot d_{t-1}$$

y_t^p is the Non Financial Permanent Income which is already exogenous

y_t^p: Constant level of Income that if provided in all future periods amounts in PV to expected income $\sum_{j=0}^{\infty} \frac{y_t^p}{(1+r)^j} = \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} \Rightarrow y_t^p = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j}$

Similarly the difference between current and Permanent Income can be obtained as $y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j}$

Closed form equilibrium solution:
In equilibrium consumption is equal to permanent income minus interest on existing debt.

Use RW ($E_t c_{t+j} = c_t$) in (2.5) to factor out consumption and solve for c_t: $\quad c_t = y_t^p - r d_{t-1}$

Closed form solution for trade balance:

$tb_t = \underbrace{y_t - y_t^p}_{\text{Plausible countercyclical } tb_t} + r d_{t-1}$ $\qquad$ tb_t will be countercyclical if y^p increases by more to changes in y_t than y_t itself (i.e., if $\frac{\partial (y_t - y_t^p)}{\partial y_t} < 0$)

External debt: the economy borrows to cover deviations of current from permanent income.

In BC (2.2): $c_t + (1+r) d_{t-1} = y_t + d_t$, Subs. $c_t = y_t^p - r d_{t-1} \Rightarrow \quad d_t - d_{t-1} = y_t^p - y_t \quad (2.12)$

Current account eq: $\quad$ **Fundamental balance of payments identity**: CA = change in NFA

(def) $ca_t = tb_t - r d_{t-1}$ $\quad$ (CA is the trade balance minus interest payments on debt)

Use solution for tb_t above: $ca_t = y_t - y_t^p$, Subsb. from change in NFA (2.12): $\quad ca_t = -(d_t - d_{t-1})$

Summary: w/ $\beta(1+r) = 1$ and Quadratic utility $\quad$ (y_t is exogenous here)

$$c_t = y_t^p - r d_{t-1} \qquad (2.11)$$

$$d_t = d_{t-1} + y_t^p - y_t \qquad (2.12)$$

$$ca_t = y_t - y_t^p \qquad (2.15)$$

$$tb_t = y_t - y_t^p + r d_{t-1} \qquad (2.16)$$

where

$$y_t^p \equiv \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} \qquad (2.10)$$

and d_{-1} and the stochastic process for y_t are exogenously given.

Principle (from 2.15): Finance temporary income shocks (w/ Δca_t) and adjust (by changing c_t but not ca_t) to permanent income shocks. Note: temporary shocks change $y_t - y_t^p$, Permanent do Not change $y_t - y_t^p$

The Income Process: for the model to yield a countercyclical CA, we need a countercyclical $y_t - y_t^p$

Which will depend on the particular process followed by y_t.

Three processes are analyzed: $\quad$ (1) $y_t \sim AR(1)$, (2) $y_t \sim AR(2)$, (3) $\Delta y_t \sim AR(1)$, y_t is $I(1)$

AR(1) process: $\quad$ Suppose $y_t - \bar{y} = \rho (y_{t-1} - \bar{y}) + \epsilon_t$, $\quad \rho \in (-1, 1)$

The j-step ahead forecast is $\quad E_t y_{t+j} = \bar{y} + \rho^j (y_t - \bar{y})$

Permanent income implications: Subs. forecast in (2.10),

$$(2.10): \; y_t^p = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{E_t y_{t+j}}{(1+r)^j} = \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{\rho^j (y_t - \bar{y})}{(1+r)^j} + \bar{y} \cancel{\frac{r}{1+r} \sum_{j=0}^{\infty} \frac{1}{(1+r)^j}}^{\frac{1+r}{r}} \Rightarrow y_t^p - \bar{y} = (y_t - \bar{y}) \frac{r}{1+r} \underbrace{\cancel{\sum_{j=0}^{\infty} \left(\frac{\rho}{1+r}\right)^j}}_{\frac{1+r}{1+r-\rho}}$$

$$\Rightarrow y_t^p - \bar{y} = \frac{r}{1+r-\rho} (y_t - \bar{y})$$

Slope $\begin{cases} \approx \frac{r}{1+r} & \text{if } \rho \to 0 \\ \approx 1 & \text{if } \rho \to 1 \end{cases}$ $\quad$ If y is very persistent most of the movements in endowment are reflected in Permanent Income

Key variable: $y_t - y_t^p = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y})$, Slope $= \frac{1-\rho}{1+r-\rho} > 0 \;\; \forall \rho \Rightarrow y_t - y_t^p$ is Procyclical

Then, the model predicts that the trade balance (tb) is procyclical. Which is counterfactual.

Consumption: by (2.11), (2.17) $c_t = y_t^p - rd_{t-1} = \bar{y} + \frac{r}{1+r-\rho}(y_t - \bar{y}) - rd_{t-1}$ $\begin{cases} \text{If } \rho = 0 \;\; c_t \text{ adjusts} < 1 \text{ w/ } y_t \\ \text{If } \rho \approx 1 \;\; c_t \text{ adjusts 1 to 1 w/ } y_t \;\; (\text{w/ } \rho = 1 \; y_t = y_t^p) \end{cases}$

Debt: by (2.12) and the expression for $y_t - y_t^p$ above: $d_t = d_{t-1} + (y_t - y_t^p) = d_{t-1} - \frac{1-\rho}{1+r-\rho}(y_t - \bar{y})$

If $\rho = 0$ debt decreases 1 to 1 w/ income. The country saves most of the temporary income to smooth consumption

If $\rho \approx 1$ debt is unchanged. (Income will be higher in the future given the permanent shock $- \Delta y_t -$, then there's no need to save.)

Current Account: by (2.15) and the eq. for $y_t - y_t^p$ above: $ca_t = y_t - y_t^p = \frac{1-\rho}{1+r-\rho}(y_t - \bar{y})$ $\begin{cases} \text{If } \rho \approx 0 \;\; ca_t \uparrow, \text{ Country saves temporary Income} \\ \text{If } \rho \approx 1 \;\; ca_t \text{ won't change} \end{cases}$

Conclusion: If $y_t \sim AR(1) \Rightarrow$ the output increase is expected to die out ($\uparrow y_t$ is temporary)
(hence future income is expected to be lower than current y_t)

$\Rightarrow$ Households will save extra income to smooth consumption $\Rightarrow \uparrow ca, \uparrow tb$
(Counterfactual)

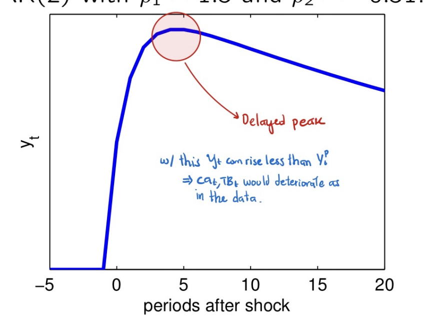
$\therefore$ We need to consider an income process that generates expectations of even higher income in the future, so that consumption (when smoothed) increases by more than output, and then agents get indebted ($\downarrow ca_t, \downarrow tb_t$)

Alternative Process: $y_t \sim AR(2)$ $\quad y_t = \bar{y} + \rho_1(y_{t-1} - \bar{y}) + \rho_2(y_{t-2} - \bar{y}) + \epsilon_t$

Process w/ hump shaped Imp. Response

Impulse Response of Endowment

AR(2) with $\rho_1 = 1.5$ and $\rho_2 = -0.51$.

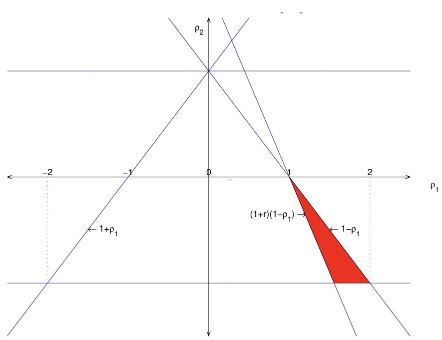


Here the peak of the output occurs several periods after the shock.

The current level of income may rise less than permanent income

Then, TB, and CA will deteriorate on impact.

We need to ensure stationarity of the process and that future income will be higher than current.
This is achieved by choosing the parameters in the red area of this plot:



Need $\rho_1 > 1$ (Future income higher than current)

$\rho_2 < 0$ (to ensure stationarity; also ρ_2 should not be too negative so it does not generate future decreases in income)

Summary of adjustment with AR(2) income

In response to a positive income shock in period t:

- y_t^p increases
- By (2.11): $c_t = y_t^p - rd_{t-1}$, hence c_t procyclical on impact.

and provided $\rho_2 > (1+r)(1-\rho_1)$

- $y_t^p - y_t$ increases,
- By (2.12): $d_t = d_{t-1} + y_t^p - y_t$, hence d_t increases.
- By (2.15): $ca_t = y_t - y_t^p$, hence ca_t countercyclical
- By (2.16): $tb_t = y_t - y_t^p + rd_{t-1}$, hence tb_t countercyclical

There are plausible conditions for the model to depict a countercyclical adjustment of the CA

Then, we do not need non-stationarity in income have such adjustment.

Non-Stationary Income Process: $\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t \quad \Rightarrow E_t \Delta y_{t+j} = \rho^j \Delta y_t$

In that case the permanent income increases more than the current one if $\rho > 0$.
Then we would obtain the countercyclical adjustment.

In general $y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j}$, Subst. $E_t \Delta y_{t+1}$ from above. Then, $y_t - y_t^p = -\frac{\rho}{1+r-\rho} \Delta y_t$

as long as $\rho > 0$, slope $< 0 \Rightarrow y_t^p - y_t$ increases

Testing the Intertemporal Approach to the Current Account

Hall (1978) initiates a literature testing the random walk hypothesis for consumption implied by the PIH (PIH: permanent income hypothesis).

Camber (1987) test predictions of PIH for savings.
Nason and Rogers (2006) test the predictions for the Current Account.
(In this model savings and CA are equal since there is no investment)

Combine (2.15) $ca_t = y_t - y_t^p$, (2.27) $y_t - y_t^p = -\sum_{j=1}^{\infty} \frac{E_t \Delta y_{t+j}}{(1+r)^j} \Rightarrow ca_t = y_t - y_t^p = -\sum_{j=1}^{\infty} (1+r)^{-j} E_t \Delta y_{t+j}$

Country runs:
Surplus if PV(Income) < 0
Deficit if PV(Income) > 0

Testable restriction: is the CA equal to the negative of the PV of income?
Both variables are observed and r can be calibrated from the literature.

$ca_t = -PV_t$ Income decreases
Approximated w/ VAR in Δy_t and ca_t given calibrated r

Estimate $x_t = D x_{t-1} + \epsilon_t$ w/ $x_t = [\Delta y_t \;\; ca_t]'$

Relation w/ PV of Income:

Forecast of x_{t+j} given info H_t is: $E_t[x_t | H_t] = D^j x_t$

Sum the forecasts: $\sum_{j=1}^{\infty} (1+r)^{-j} E_t[x_{t+j} | H_t] = [I - \frac{D}{1+r}]^{-1} \frac{D}{1+r} \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$

Pre multiply by $[1\; 0]$ to isolate income: $\sum_{j=1}^{\infty} (1+r)^{-j} E_t[\Delta y_{t+j} | H_t] = \underbrace{[1\; 0][I - \frac{D}{1+r}]^{-1} \frac{D}{1+r}}_{-F} \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix} \Rightarrow F \cdot x_t = -\widehat{PV}_t$ of Income (decreases)

then we want to test $H_0: F = [0 \;\; 1]$ (H_0: 2 restrictions imposed on a function of the coefs. in D)

Nason and Rogers (2006) carry out this test based on a VAR(4) for Canada (1963Q1-1997Q4). They assume a r = 0.037 per year. After computing the associated Wald statistic they find that the **null hypothesis is rejected**. (The data does not support the Intertemporal approach to the CA).

Conclusions

The Intertemporal Approach to the CA is not good for explaining the CA, TB. Cannot explain countercyclical patterns, or yields predictions about the PV of income that don't hold.

For AR(1) income processes the model predicts a procyclical CA and TB

For AR(2) income processes (or non stationary) the CA and TB can be countercyclical. But the model yields predictions about the relationship between the CA and the PV of future income (that the CA is equal to minus expected future income changes) that are poorly supported by the data.

Then, we need to include richer sources of dynamics and mechanisms of propagation.

Summary Ch 3 — An Open Economy with Capital

Capital is introduced as endowment model fails to explain a countercyclical trade balance (tb_t) and because we want to endogenize the output (and cycles)

Simplifying assumptions: Perfect foresight, no depreciation, $\beta(1+r)=1$ (With no uncertainty and such r, we end up with a constant consumption path, Ct = C for all t)

w/ K the tb_t can become countercyclical

w/ AR(1) income: given persistent productivity shocks the MPK is expected to be high in the future
$\Rightarrow$ Agents Increase Investment $\Rightarrow \uparrow$ Demand, ($\uparrow(C+I)$ relative to Output) $\downarrow$ Savings $\Rightarrow \downarrow TB_t$

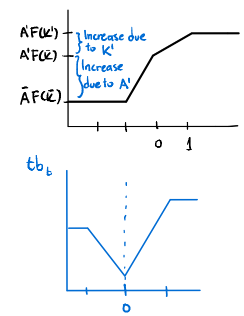
Now the model is: $\max_{c_t, d_t, k_{t+1}, i_t} \sum \beta^t u(c_t)$ s.t. $c_t + i_t + (1+r)d_{t-1} = y_t + d_t$ w/ $y_t = A_t F(k_t)$ and $k_{t+1} = k_t + i_t$ (as before $\lim_{j\to\infty} \frac{d_{t+j}}{(1+r)^j} \le 0$)

We will use Intertemporal BC approach to characterize consumption: $c_t = \frac{r}{1+r}\sum_{j=0}^{\infty} \frac{A_{t+j}F(k_{t+j}) - (k_{t+j+1} - k_{t+j})}{(1+r)^j} - r d_{t-1}$ (Now Consumption reflects Investment expenditure)

i_t

$tb_t = y_t - c_t - i_t$ ($\Delta c_t > \Delta y_t$ no longer required for ∇tb_t)

What is new that allows Tbt to fall? -> Yt is expected to increase in the future -> agents invest more



In contrast:
with permanent k_t doesn't increase
$\Rightarrow Tb_t$ improves by $\uparrow y$ (relative to C)

Two Principles:

I. w/ More persistent shocks $\rightarrow$ higher deterioration of TB_t

$\hookrightarrow$ with permanent shock TB_t deteriorates at the period of the shock!
(transitory) (improves)

II. w/ higher K Adj. costs $\longrightarrow$ Smaller TB_t deterioration Agents will smooth the ΔI ($\Rightarrow TB_t$ won't be lowered by too much)

w/ Adj. Costs: ΔInvestment is spread out (smoothed) $\Rightarrow$ lower $t=0$ increase in: permanent income, consumption $\Rightarrow$ Muted Instantaneous Tb_t response. Costs $= \frac{1}{2}\frac{i_t^2}{k_t}$

Budget Constraint becomes: $c_t + i_t + \frac{1}{2}\frac{i_t^2}{k_t} + (1+r)d_{t-1} = A_t F(k_t) + d_t$

(UMP) Lagrangian: $\mathcal{L} = \sum_{t=0}^{\infty}\left\{U(c_t) + \lambda_t\left[A_t F(k_t) + d_t - c_t - i_t - \frac{1}{2}\frac{i_t^2}{k_t} + q_t(k_t + i_t - k_{t+1})\right]\right\}$ q_t: Tobin's q (LMult of K-dynamics is $\lambda_t q_t$)

New Euler equation: Return on Debt = Return on Physical Capital
(of q_t cap. units) (extra output + New Price of K + Reduction in costs)

New Ct dynamics with intertemporal BC now accounts for adjustment costs on the RHS $-\frac{1}{2}(i_{t+j}/k_{t+j})$
(as before) Consumption = Permanent Income = investment income (debt) + Non Financial Permanent Income

combine FOC to get dynamics of K stock in terms of prices, quantities:

$$k_{t+1} = q_t k_t$$
$$q_t = \frac{A_{t+1}F'(q_t k_t) + (q_{t+1}-1)^2/2 + q_{t+1}}{1+r}$$

$\xrightarrow{\text{log-linearize}}$

$$\hat{k}_{t+1} = \hat{q}_t + \hat{k}_t$$
$$(1+r)\hat{q}_t = -r_{\epsilon F'}\hat{k}_{t+1} + \hat{q}_{t+1}$$

SS: $q=1$ $r = \bar{A}F'(k)$

in matrix form: $\begin{bmatrix}\hat{k}_{t+1}\\ \hat{q}_{t+1}\end{bmatrix} = M\begin{bmatrix}\hat{k}_t\\ \hat{q}_t\end{bmatrix}$ w/ $M = \begin{bmatrix}1 & 1\\ r_{\epsilon F'} & 1+r+r_{\epsilon F'}\end{bmatrix}$

the solution will satisfy: $\lim_{t\to\infty}\begin{bmatrix}\hat{k}_t\\ \hat{q}_t\end{bmatrix} = \lim_{t\to\infty} M^t\begin{bmatrix}\hat{k}_0\\ \hat{q}_0\end{bmatrix}$

($\exists$ unique q_0 s.t. equilibrium converges to SS given k_0)

For a unique solution 1 eigenvalue of M lies outside unit circle and 1 inside

Let these eigenvalues be λ_1, λ_2. The dynamics are found as: $\hat{k}_t = \lambda_2^t \hat{k}_0$
(also $\hat{q}_0 = -(1-\lambda_2)\hat{k}_0$) $\hat{q}_t = -(1-\lambda_2)\lambda_2^t\hat{k}_0$

$\therefore$ i) Unique saddle path stable eq. exists (in neighborhood around (q^{ss}, k^{ss}))

ii) adj. to permanent increase in productivity induces k to converge from below (K increases) and Tobin's q from above (q decreases until hitting SS)

iii) Increase in capital is spread out in many periods $\Rightarrow$ Investment is positive and because ΔK is independent of the size of adj. costs: Principle II Follows (or *)

*: Capital adj. costs dampen tb_t deterioration in response to a permanent productivity increase