


ECON 5322

Macroeconomic Theory for Applications

Topic 1: Course Overview and Introduction

Part 1: Macroeconomic data facts

1. Course Logistics (syllabus and lectures plan)
2. Please note HW0 on course website 
3. Basic Business Cycle Facts
4. A Brief History of Modern Business Cycle Theory

About me: Camilo Granados

- Assistant Professor of Economics – UT Dallas, EPPS
- Background: Ph.D. in Economics (U. Washington), B.S. and M.S. Economics and Postgraduate Diploma in Statistics (U. Nacional, Colombia)
- Born in Colombia
- Worked in a Central Bank for 13 years (09-21²), including a leave for the Ph.D.
 - Economist, Research Economist
- Teaching experience:
 - UTD: international finance (ugrad), macroeconomics (MS), international finance and macroeconomics (PhD)
 - UW: advanced macro, international finance, business finance, intro to macroeconomics, MBA microeconomics, elements of statistics
 - Before the PhD: Advanced Econometrics, Advanced Microeconomics, Game Theory, Principles of Economics.

General Teaching Philosophy

- Train students how to think about economics
- Ultimately, it is not just what you know but also how you think - synthesize and process information - that matters
- Lectures won't follow textbooks exactly; they draw out important concepts from some books and rely on study notes.
- Lecture slides available ahead of lecture to facilitate note-taking.
 - My advice: download them and take notes on top during class
- Slides with annotations are posted AFTER lecture
- Aim for more interaction - please "raise hands"

Requirements: See Syllabus (read it closely, It contains our “playing rules” and I will assume it is common knowledge to everyone)

Textbooks: Romer, Advanced Macroeconomics, 5ed., McGraw Hill and Chugh, Modern Macroeconomics, 2015.

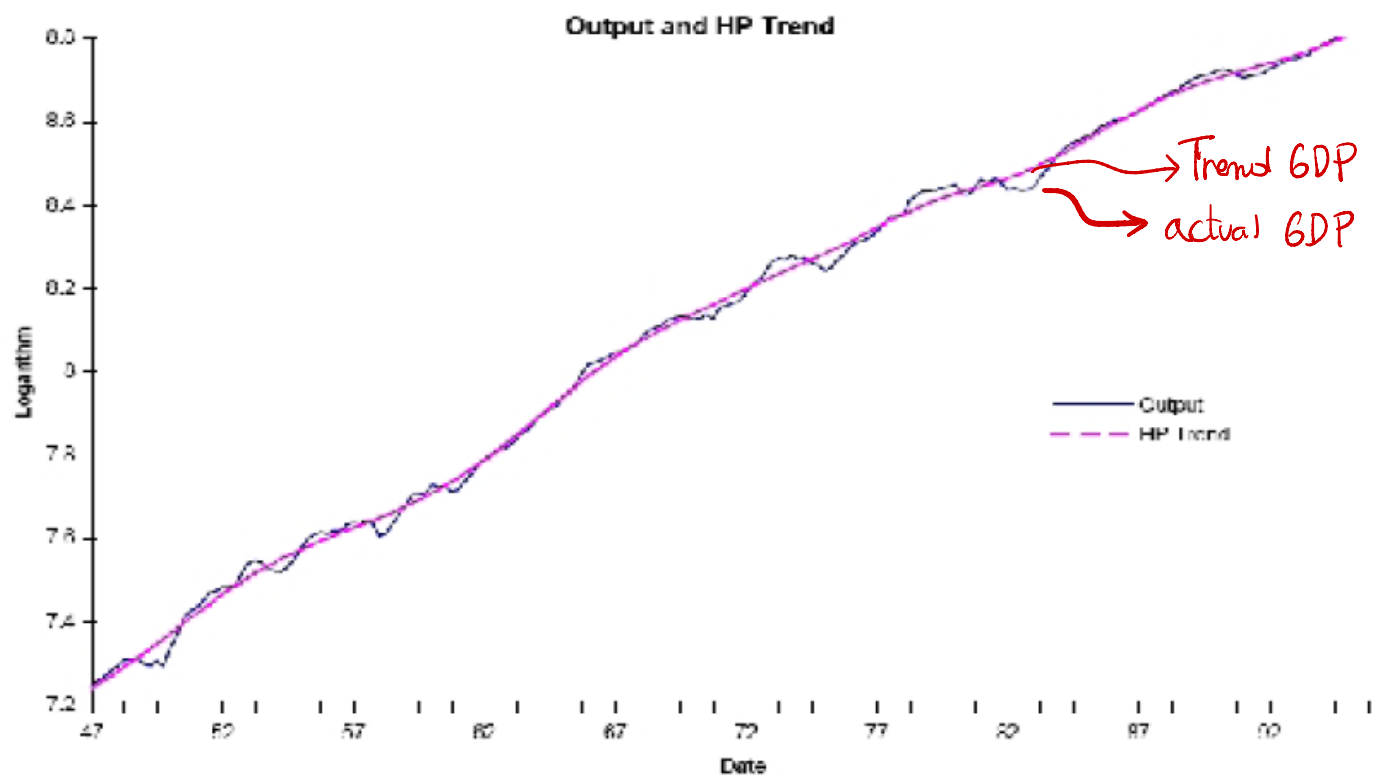
(we’ll use them for some end of chapter problems but you can do the whole course with the slides and study notes only)

This course is not easy: heavy on math

Given that, I try to help:

- Most of the grading weight is put on take-home assignments rather than in-lecture tests
- The heavy lifting is put on the homework and practical projects
- If I see effort I am happy to adjust the grading weights along the way

Some Business Cycle Facts (for the U.S. 1947:1 – 1996:4)



Decompose "Trend" vs. "Cycles" in a time series

$$y_t = y_t^T + y_t^C \text{ (e.g., with filters)}$$

$$GDP_t = GDP_t^{\text{Trend}} + \underbrace{GDP_t^{\text{Cycle}}}_{\text{wiggles or fluctuations around trend}}$$

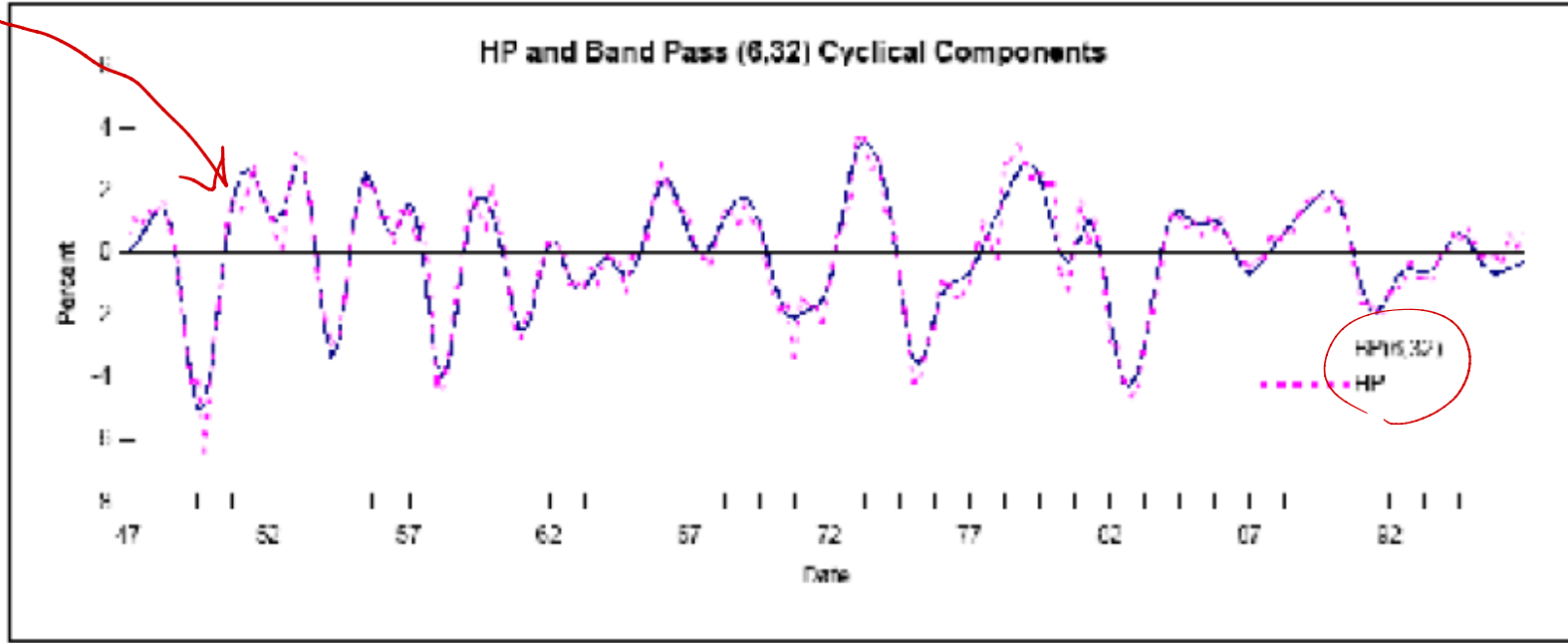
$$y_{t}^{cycle} = y_t - y_t^{Trend}$$

Hodrick-Prescott filter → hpfilter.m ($\lambda = 1600$ Quarterly data)
 Other ways: band pass filter, Christiano Fitzgerald;
 Linear trend, Linear-quadratic trend

Cyclical Component

$$\hat{y}_t = \alpha + \beta_1 t + \beta_2 t^2 + \epsilon_t$$

↳ high freq fluctuations
 ↳ cycle



Common **filters**: linear, Hodrick-Prescott (HP), Bandpass

Intuitively: a way to extract the long-run growth portion or the “low frequency” components of the data

NBER → are we in a recession?

Business Cycle Dating

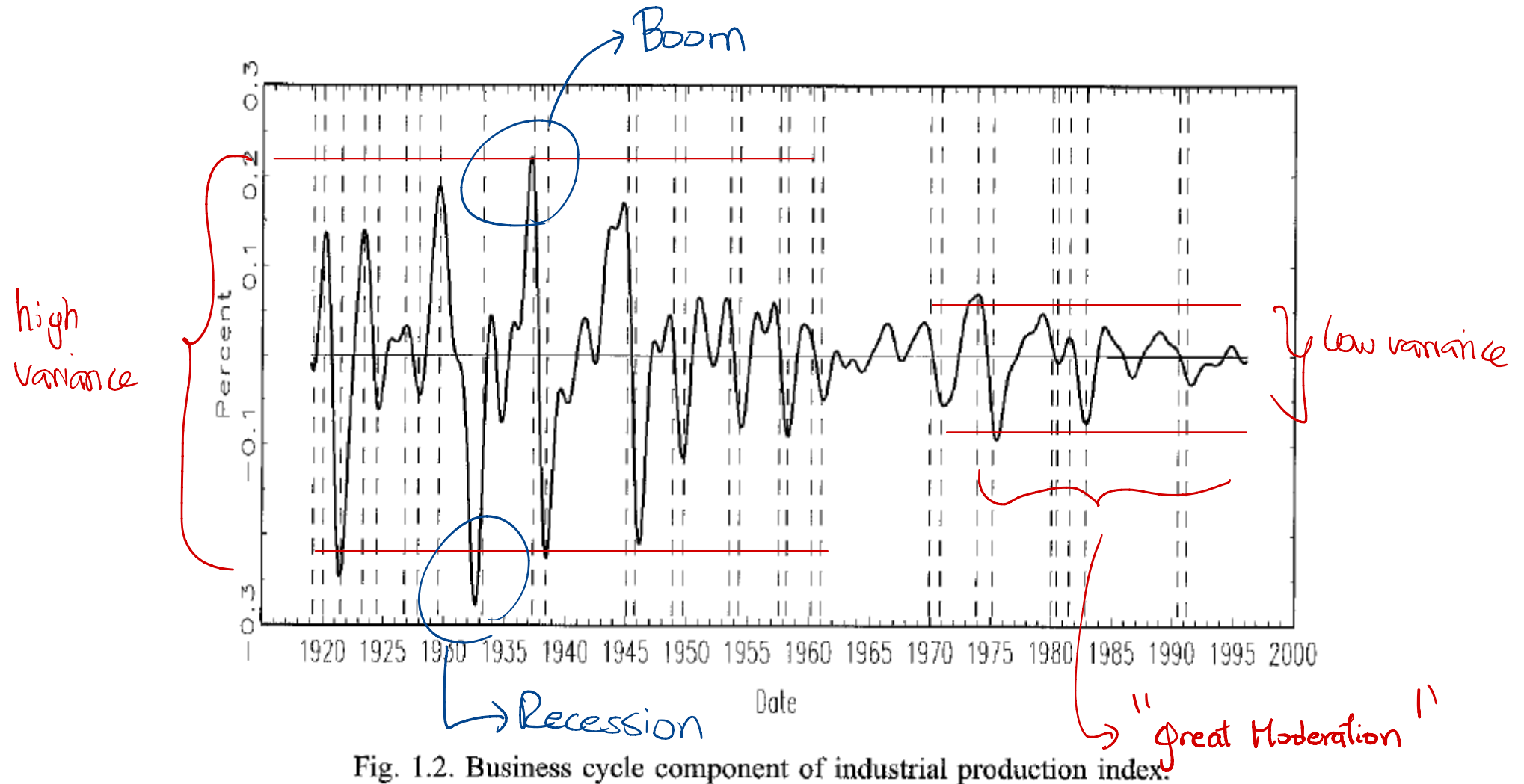


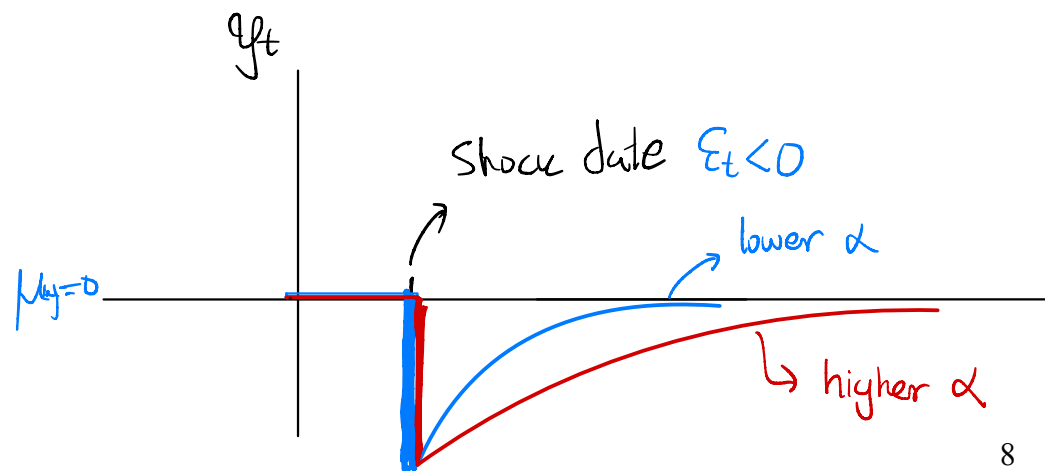
Fig. 1.2. Business cycle component of industrial production index.

Some Stylized Facts about the U.S. Business Cycles:

What do we look for?

1. Volatility/amplitude of fluctuations → Variance of a cycle
2. Comovements → Covariances & Correlations w/ other macro variables
3. Persistence/lead and lag patterns → Autoregressive patterns

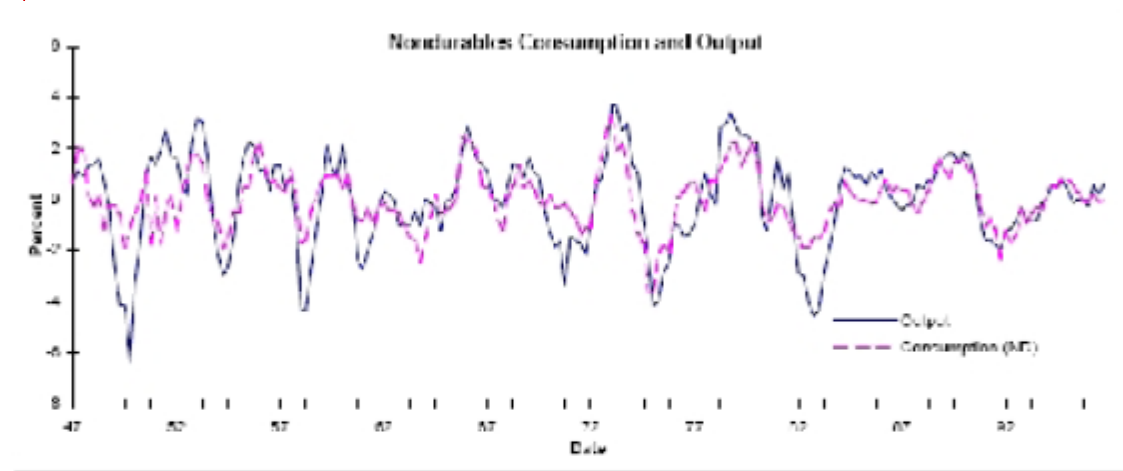
$$\text{AR process: } y_t = \mu_y + \alpha y_{t-1} + \varepsilon_t$$



$$y_t = \underline{C_t} + I_t + G_t + NX_t \quad \circ \text{ (Closed economy)}$$

Nondurable consumption

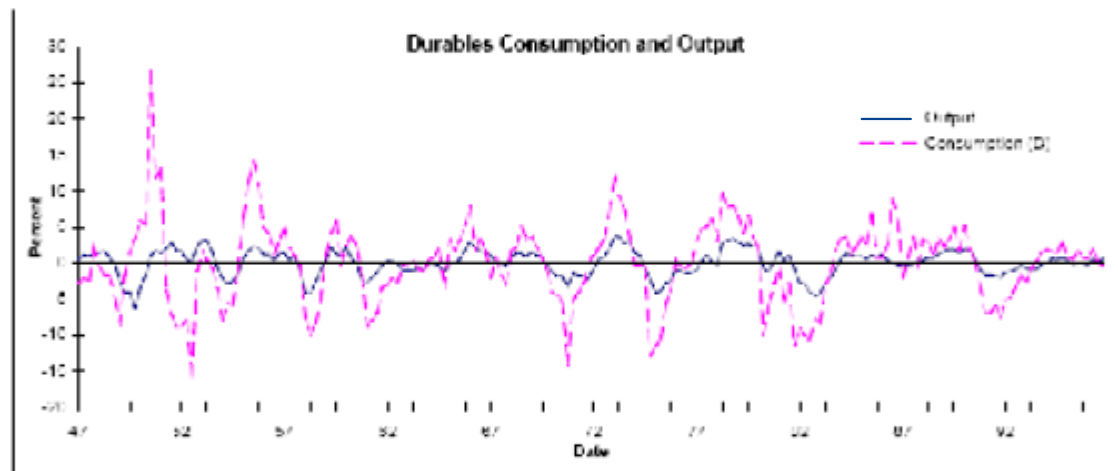
→ Non durable
→ Durable



$$\sigma_{C, \text{nondurable}} < \sigma_y$$

* Less volatile than output

Consumer durables



* More volatile than output

$$\sigma_{C, \text{non-durable}} < \sigma_y$$

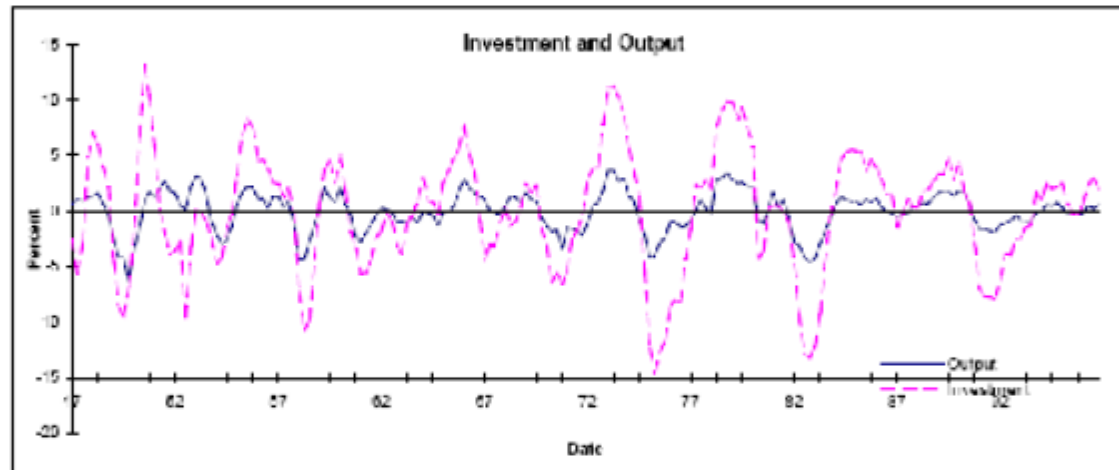
$$\sigma_{C, \text{durables}} > \sigma_y$$

Investment

$$\sigma_{C, \text{non-durable}} < \sigma_y$$

$$\sigma_{C, \text{durables}} > \sigma_y$$

$$\sigma_I > \sigma_y$$



* 3 times more volatile than output

$$y_t = C_t + I_t + G_t$$

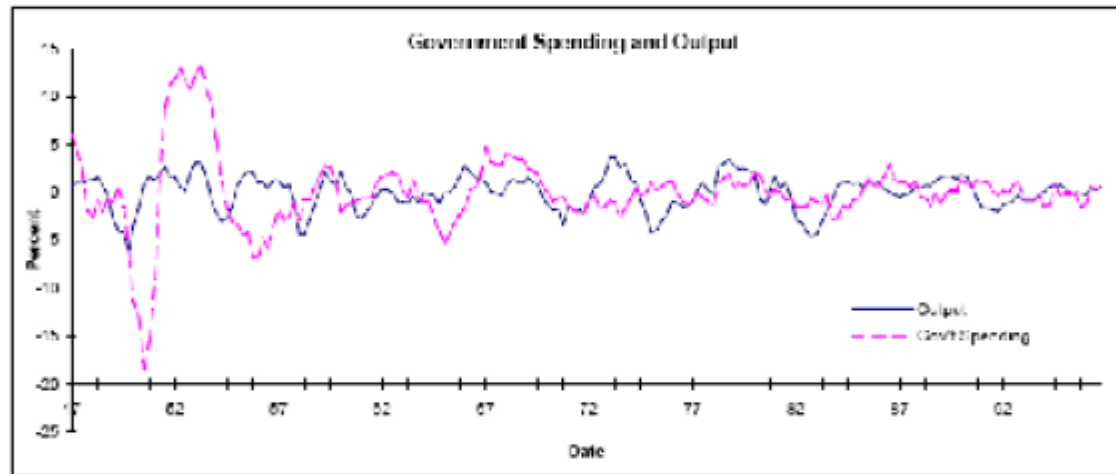
Government expenditures

$$\sigma_{C, \text{non-durable}} < \sigma_y$$

$$\sigma_{C, \text{durable}} > \sigma_y$$

$$\sigma_I > \sigma_y$$

$$\sigma_G < \sigma_y$$



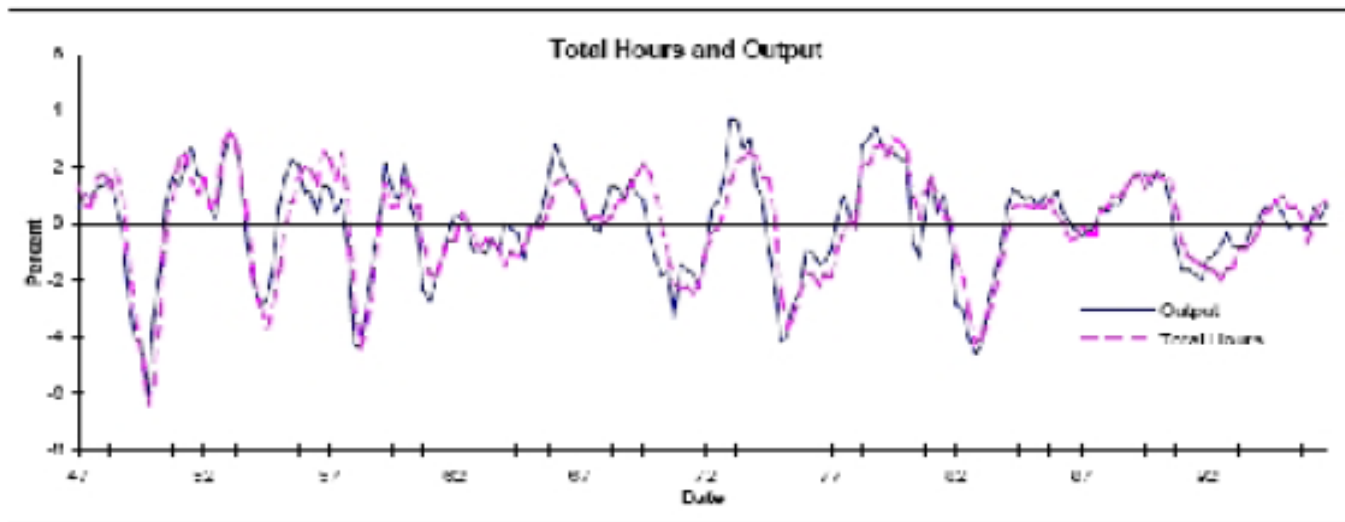
* Less volatile than output

Total hours worked

$$(N \times \text{hrs per worker})$$

↓

$$(\text{Number of workers})$$



$$G_{\text{Total hours}} \approx G_y$$

highly correlated with
output

- About as volatile as output
- Business cycle is most clearly manifested in the labor market

Employment

↳ # of workers (N)



$$\sigma_N \approx \sigma_Y$$

- as volatile as output

Hours worked

$$\sigma_{\text{Hours}} < \sigma_y$$



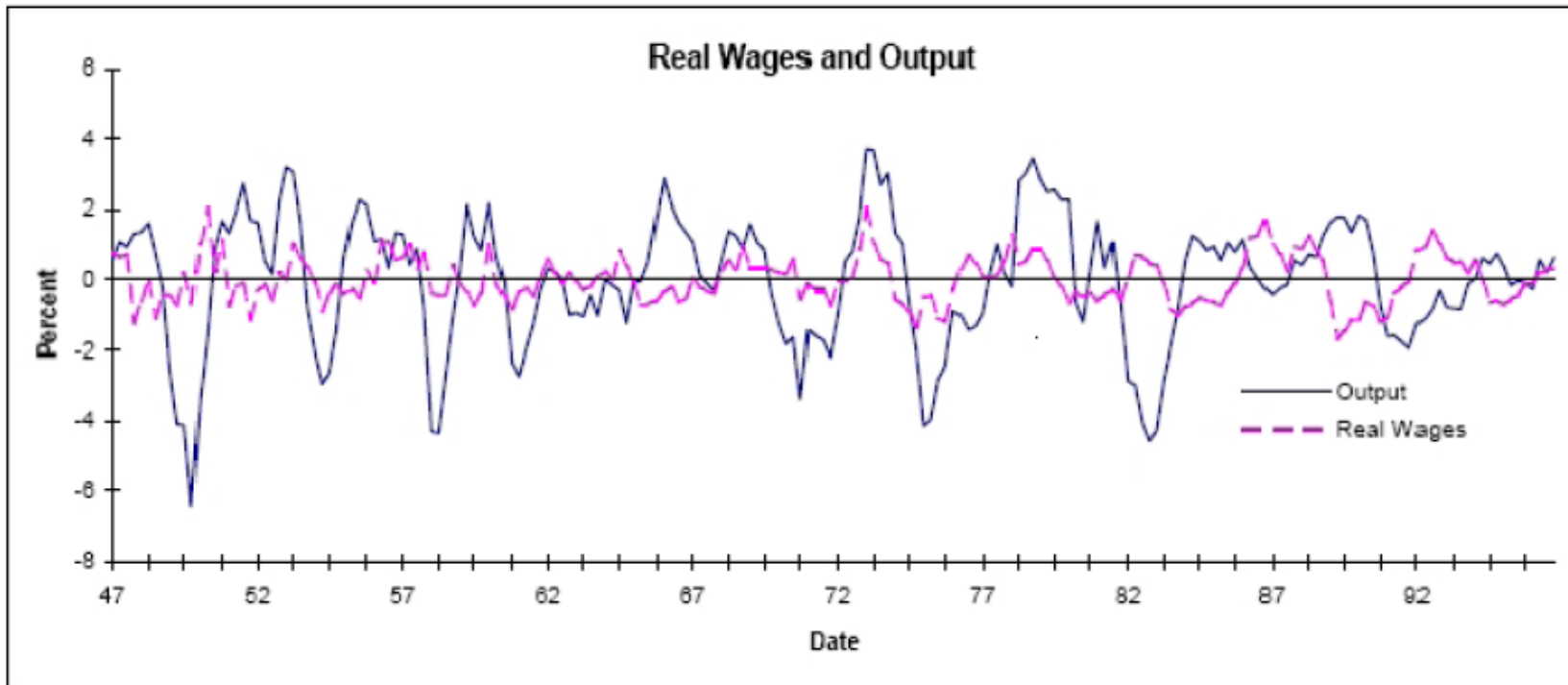
- Hours **per worker**: Much less volatile than output
- Most variation in total hours stems from changes in employment, rather than adjustment in hours worked per employee.

$$\text{Real wage} = \frac{W}{P}$$

→ Nominal wage
→ Prices level

$$\text{Real Wage} \sim \text{MPL} \left(\frac{\partial Y}{\partial L} \right)$$

$$\log \text{ real wage} = \log W - \log P$$



- Much less volatile than output
 - Slightly **PRO-cyclical** (correlation = 0.14) \Rightarrow important fact
(or even a-cyclical)
- ⊕ Correlated w/ GDP cycle

Are we still going through a great moderation?
(HW1)

An Obvious Question

- What do the US business cycle statistics look like post-1999 & post-2007?
Are the patterns described above still true? How about other countries?
- How to produce the basic business cycle statistics: discussed in the first two sections of King and Rebelo (1999) Handbook chapter

↳ Uribe & Schmitt (2017)

Data Sources:

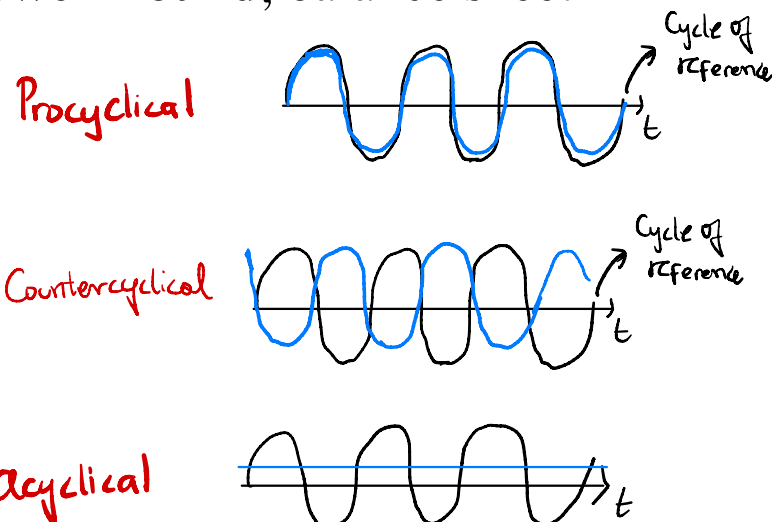
- for the US: FRED
 - <https://fred.stlouisfed.org/>
- international macro: IMF-International Financial Statistics (IFS)

→ HW1

Can you see in the data the following view?

- Old Macro: Analyzes pre- versus post- 1984:Q4. Great moderation
- New Macro: Analyzes pre- versus post- August 2007. Global Finance Crisis
Pandemic crisis / Inflation Surge
 - End of the **Great Moderation**
 - Downturn precipitated by disruption of **Financial Intermediation**
 - **Unconventional Monetary Policy and Zero Lower Bound**, balance sheet management, macro-prudential policy...

- COVID-19 macro dynamics Supply or demand shock? both?
How transitory?
 - New inflation dynamics Inflationary consequences?



As we go through models after models after models*... ALWAYS ask yourself:

1. What is the Motivation behind the model?
2. What is the **Economic intuition**?
3. What is the relevant Technique/Tool to pick up?
4. What does the Data say?
5. What are some Alternatives to model or test the same phenomenon?

=> Learn not (just) their thoughts, but how they THINK (how to approach and formalize the issue at hand)

Key Questions (Extra Credit)

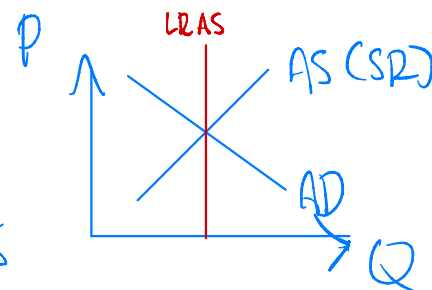
- What are some “stylized facts” about US Business cycle dynamics up?
- What is Neoclassical Synthesis?
- ... more to follow

Part 2: From IS-LM Neo Classical Synthesis to New Synthesis

A. Some course logistics – **important dates: posted on course website!**

B. Brief History of Modern Macro

- The Neoclassical Synthesis → IS-LM → AD-AS
- The Breakdown of the Consensus → 1970's stagflation ($\uparrow P, \downarrow Q$)
 - o Price Adjustment: Phillips-Solow-Samuelson vs. Friedman-Phelps
- Is the Long-Run Phillips Curve Really Vertical?
- Rational Expectations Revolution
- Main Approaches to modeling Aggregate Supply since then
- The New-Neoclassical Synthesis → RBC toolkit + Keynesian market frictions ideas



Housekeeping:

- **First midterm** is done after **Topic 2**
- The **6 Homework assignments** are submitted in class
 - First Assignment (HW0) due next week: Get full credits just by submitting (only for HW0)
- A **Final Exam**, at the end of the semester
- Last week of class: **group project presentations**
- Reminder: **Extra credit option**: submitting answers to “Key Questions” at the end of each lecture (before the start of the next)

A VERY Brief History of Macro:

- Neoclassical Synthesis (IS-LM: AD-AS → Intermediate macro)
- The Breakdown of the Consensus + the Rational Expectations Revolution
- Main approaches to modeling Aggregate Supply in the 80s & 90s
- “New” Neoclassical Synthesis → { Old lessons from initial synthesis
+ Rational Expectations
+ Intertemporal Decision making
- Post-2008: Financial Friction, “Unconventional” Monetary Policy...etc.
- Now?

I. **Neoclassical Synthesis**: Consensus in the 1960's (Review of intro to macro)

a. **Aggregate Demand**: from IS-LM framework: **goods market** and **money market** equilibria, and Walras Law implies **asset market**

clearing → (Supply = Demand)

① Goods market: $Y = C(Y-T) + I(r) + G + NX$

Annotations: $i - \pi$ (with $\log P_t - \log P_{t-1}$ above it), r (with $r + \pi$ below it), NX (with "ignore for now" next to it). A red '+' is under $C(Y-T)$ and a red '-' is under $I(r)$.

② Money market: $\frac{M^s}{P} = L(i, Y)$

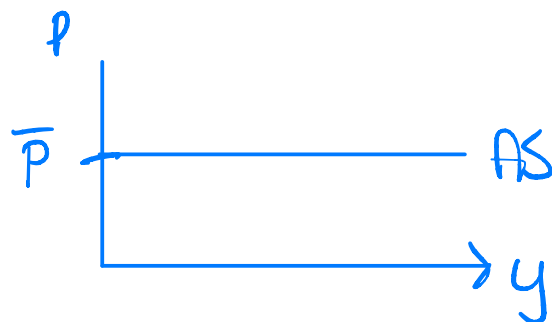
Annotation: $r + \pi$ below i .

+ Walras Law: If $N-1$ markets are in equilibrium, the N -th remaining market is in equilibrium.

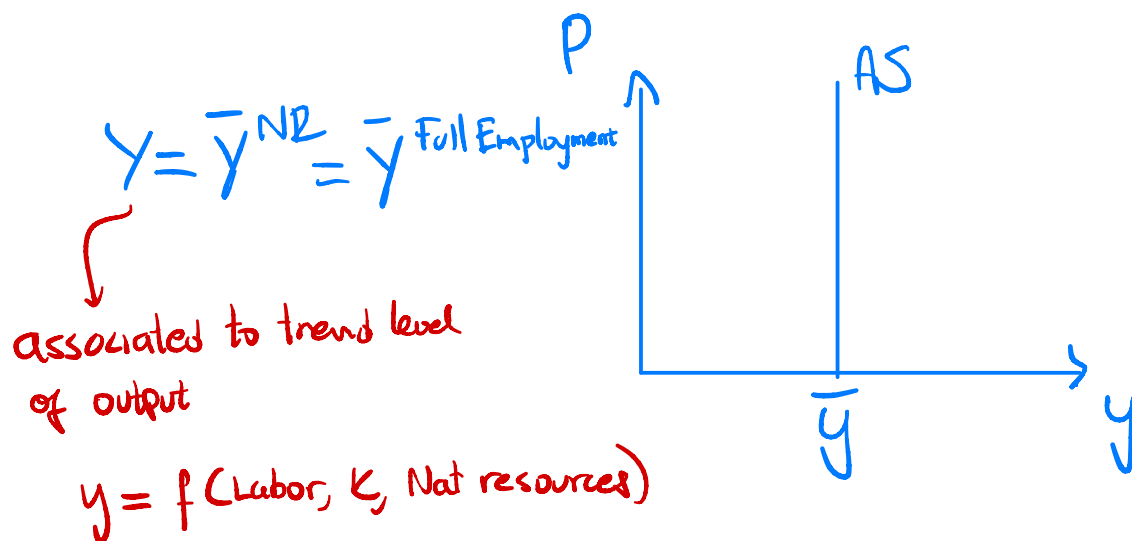
⇒ Obtain Aggregate Demand (from IS-LM equilibrium)

b. **Aggregate Supply:** Keynesian vs. Classicalist: Are prices sticky?

i) Keynesian Nominal rigidity (sticky prices/wages) => Short-run

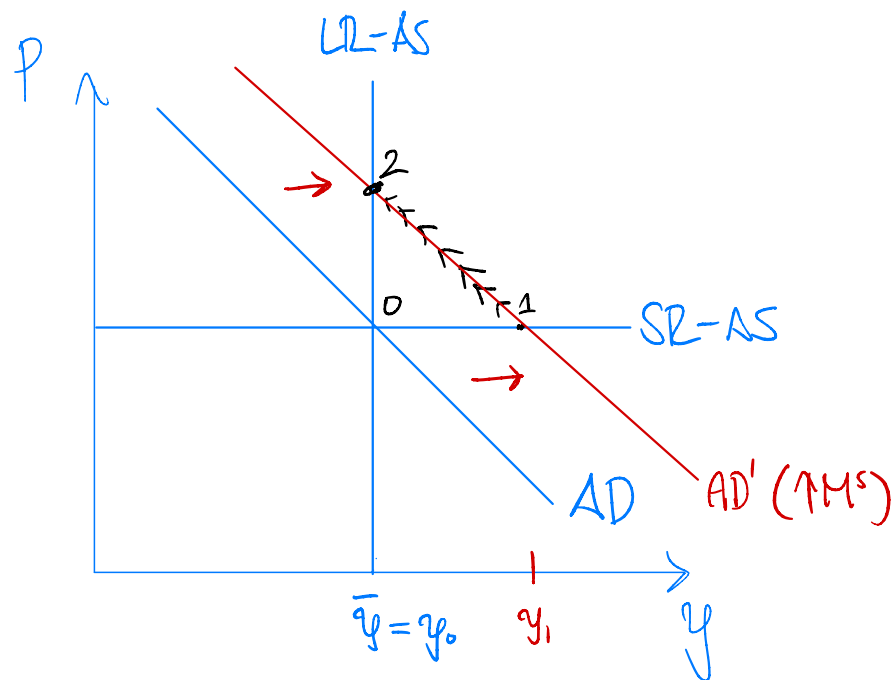


ii) Natural Rate Hypothesis (prices/wages fully flexible) & **monetary neutrality** => Long-run

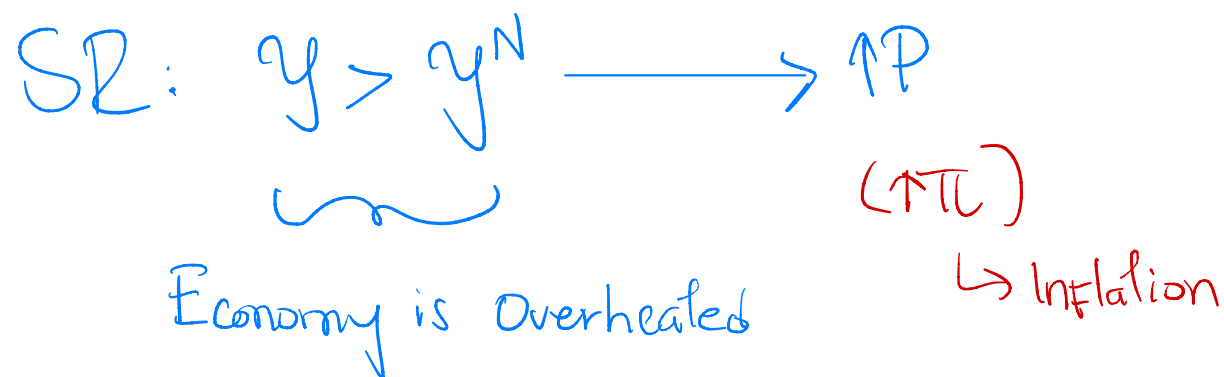


Neoclassical Synthesis: put relationships above together,

- with 1) and 2) giving AD
- i) for Short-Run AS
- ii) for Long-Run AS



How do we adjust from SR to LR?



c. linking SR and LR with **price-adjustment dynamics**: the Phillips

Curve:

$$\pi = \alpha (Y - Y^N) = -\beta (u - u^N)$$

$\alpha > 0$
 $\beta > 0$
 $u - u^N$ \rightarrow unemployment rate

$\left\{ \begin{array}{l} \text{Corr}(y, N) > 0 \\ \text{Corr}(y, u) < 0 \end{array} \right.$

How well did this “model” do?

- Data in the 50’s-60s supported the above => tradeoff between π and u , providing **scope for policy actions**
- At the applied level: refinements of above
 - large-scaled models: “MPS”, Harvard, Fed models with several hundred of equations
 - these models are based on empirically observed relationships (between output and consumption, money demand, in unemployment... etc) → Correlations
 - the aim of these models was to predict the effects of policies
 - they were pretty successful at it until the 1960s
- Bob Solow: “Macroeconomics is finished” (as in done/completed!)

The Breakdown of the Consensus in the early 1970s

Stagflation:
Simultaneous increase
of π and u

a. Empirically:

Models couldn't explain the simultaneous rising inflation AND unemployment in the 1970s: Vietnam War, $G \uparrow$, $M^S \uparrow \Rightarrow \pi \uparrow$ but no $u \downarrow$

b. Theoretically:

- Friedman (1968), Phelps (1968): Phillips' curve cannot be right!

Violation of the **Natural Rate Hypothesis**: Long run unemployment should NOT depend on the average rate of money growth, i.e. **What if Fed changes the money growth rate?? Say from 0% to 5%, $\Rightarrow \pi = 5\% \Rightarrow u \downarrow$ and $Y \uparrow$ in the LR!**

\Rightarrow Expectation-augmented Phillips Curve

$$\pi = -\beta(u - u^N) + \gamma E\pi$$

Compatible, if

As policy changes from 0 to 5% (money growth)
 $E[\pi]$ increases from 0 to 5% (assumes $\gamma \approx 1$)

- Lucas Critique ('73) and the **Rational Expectation** revolution
- When evaluating policy, need to consider the feedback with expectations: if policy maker changes the rule, public expectation will adjust as well, so the equilibrium condition for the economy will change too.

⇒ All of the above point to the “danger” of using ad hoc, reduced-form empirical relationships with no “micro-foundation”!

Correlations

Tom Sargent, “Macro is finished”... (as in “destroyed”)

20+ years of confusion and division to follow....

Lucas Critique ('73) and the Rational Expectation Revolution

- Expectation-augmented Phillips curve:

$$\pi = -\beta(u - u^N) + \gamma E\pi$$

→ Feedback of expectations into π

→ Expected Inflation

Or:

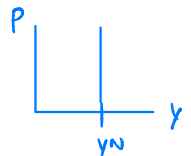
$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$

Note: in the long-run, $\pi = E\pi$ (by definition, of LR)

Long run:

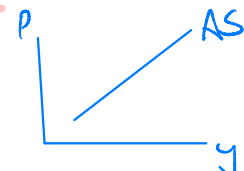
- If $\gamma = 1$, then $u = u^N$, so LR, at u^N and $Y^N \Rightarrow$ LR-AS is vertical

$$\gamma = 1 \Rightarrow \pi = -\beta(u - u^N) + \gamma \pi \Rightarrow u = u^N \quad (\gamma = Y^N)$$



- Otherwise, u will depend on both u^N and $\pi \Rightarrow$ LR-AS is sloped (Y or u depend on prices/inflation)

$$(\gamma \neq 1, \gamma < 1) \Rightarrow \pi = \frac{-\beta}{1-\gamma} (u - u^N)$$



How to Measure/Model $E\pi$

$$\pi_t = a - bu_t + \underbrace{\gamma}_{\textcircled{1}} E_{t-1} \pi_t$$

$\overline{\gamma} \rightarrow \textcircled{2}$

- It is therefore important to know the value of γ ,
- To gauge its value: look into past data of inflation and unemployment,

BUT:

- also need to know how to measure and model $E\pi$

Lucas Critique ('73) and the Rational Expectation Revolution

- Nobel Prize 1995
- Much of what economists were doing and the policy conclusions were WRONG
 - Using a model with fixed coefficients estimated from reduced-form equations and historical data to evaluate the effects of new policy would give misleading results because **expectations need to be endogenous**
 - i.e. Changes in policy will affect expectations
- True whenever expectations are forward-looking (need not be rational)

“Rational” = model-consistent

Three Methodological Tenets for Rational Expectations¹

- Results widely applicable; use Phillips Curve as an example below

1. Partial Equilibrium.

↑ market + account by stochastic behavior of $E_{t-1}W$

Agents form expectations appropriately given the **stochastic process** generating the variables of interest. Expectations cannot be specified without first specifying the underlying stochastic process.

e.g. Given $\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$ (*), want to know γ

=> first specify how π_t is determined

e.g. π_t follows an AR(1) process

$$\pi_t = \rho \pi_{t-1} + \varepsilon_t$$

$\hookrightarrow 0 < \rho < 1$

Show (white noise process)

$$\begin{aligned} E(\varepsilon_t) &= 0 \\ E(\varepsilon_t^2) &= \sigma^2 \\ E(\varepsilon_t, \varepsilon_s) &= 0 \end{aligned}$$

¹ N.G. Mankiw

Explain Expectation Operator (and take $E_{t-1}\pi_t$ given it is AR(1))

$E[X_t]$: Expected value of X_t

$E_{t-1}[X_t]$: $E[X_t | \text{Information up to } t-1] = E[X_t | I_{t-1}]$

Properties:

① $E[\cdot]$ is a linear operation

② $E[\alpha X_t] = \alpha E[X_t]$
 α is constant

$$E_{t+1}[\pi_t] = E_{t+1}[\rho \pi_{t-1} + \epsilon_t]$$

$$E_{t+1}[\pi_t] = E_{t+1}[\rho \pi_{t-1}] + E_{t+1}[\epsilon_t]$$

$$E_{t+1}[\pi_t] = \rho E_{t+1}[\pi_{t-1}] + \underbrace{E_{t+1}[\epsilon_t]}_{=0}$$

$$E_{t+1}[\pi_t] = \rho \pi_{t-1}$$

Given the stochastic process, $E_{t-1}\pi_t = \rho\pi_{t-1}$ under Rational
Expectation

- Plug this into (*), we see that

Plug it into PC

$$\pi_t = a - bu_t + \gamma\rho\pi_{t-1}$$

- What does this say about finding γ by looking at the historical relationship b/w inflation and unemployment?

2. General Equilibrium.

The stochastic process of any variable generally depends on the entire model. To solve for expectations, one must assume that agents know and solve correctly the model of the economy.

Simple 6 Eq model (all variables in logs)

$$\textcircled{1} \quad m_t - p_t = y_t \quad (\text{AD})$$

$$\textcircled{2} \quad y_t = \alpha (p_t - E_{t-1} p_t) \quad (\text{AS})$$

$$\begin{aligned} \hookrightarrow y_t &= \alpha (\underbrace{\pi_t}_{p_t - p_{t-1}} - \underbrace{E_{t-1} \pi_t}_{E_{t-1} (p_t - p_{t-1})}) \end{aligned}$$

(solve for Y from the system : Eqs. 1-2)

$$\textcircled{1} \quad m_t - p_t = y_t \quad (\text{AD})$$

$$\textcircled{2} \quad y_t = \alpha (p_t - E_{t-1} p_t) \quad (\text{AS})$$

from (1): $p_t = m_t - y_t$

Take E_{t-1} : $E_{t-1} p_t = E_{t-1} (m_t - y_t) \quad (1')$

from (2): $E_{t-1} [y_t] = \alpha E_{t-1} (p_t - E_{t-1} p_t)$
(Take E_{t-1})

$$= \alpha E_{t-1} p_t - \alpha \underbrace{E_{t-1} [E_{t-1} p_t]}_{E_{t-1} p_t} = 0$$

$$\rightarrow E_{t-1} [E_{t-1} [X_t]] = E_{t-1} [X_t]$$

$$\Rightarrow \text{in (1')}: E_{t-1} p_t = E_{t-1} m_t$$

back in AS: $y_t = \alpha (p_t - E_{t-1} p_t)$

$$y_t = \alpha (m_t - y_t - E_{t-1} m_t)$$

y is a function of **unexpected movements** in money supply

Now, solve for y_t :

$$y_t = \frac{\alpha}{1+\alpha} (m_t - E_{t-1} m_t) \quad (\text{AS}^{\text{New}})$$

Policy Rule: Framework according to which policy is set

3. Policy Evaluation

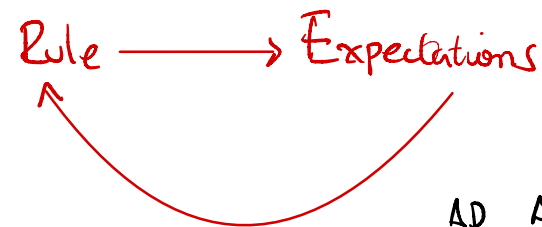
The rules governing policy are among the equations in any complete model of the economy. Because a change in a policy rule alters the stochastic processes generating many variables, it also changes the way people form expectations. Hence, when evaluating alternative policy rules, one must take account of this feedback between the policy rule and the way expectations are formed. Any policy evaluation that fails to take account of this feedback is flawed and useless.

'Lucas critique'

↳ Need to specify a rule

e.g. simple rule $m_t = \mu_A + m_{t-1} + \varepsilon_t$ (3) \Rightarrow (use it along w/ ΔD ΔS (1), (2))

$$\begin{aligned} E_{t-1} m_t &= \mu_A + E_{t-1} m_{t-1} + E_{t-1} \varepsilon_t \\ &= \mu_A + m_{t-1} \quad (\text{Plug it into } AS^{\text{New}}) \end{aligned}$$



$$y_t = \frac{\alpha}{\alpha+1} [m_t - E_{t-1} m_t]$$

$$y_t = \frac{\alpha}{1+\alpha} (m_t - E_{t-1} m_t) \quad (AS^{New})$$

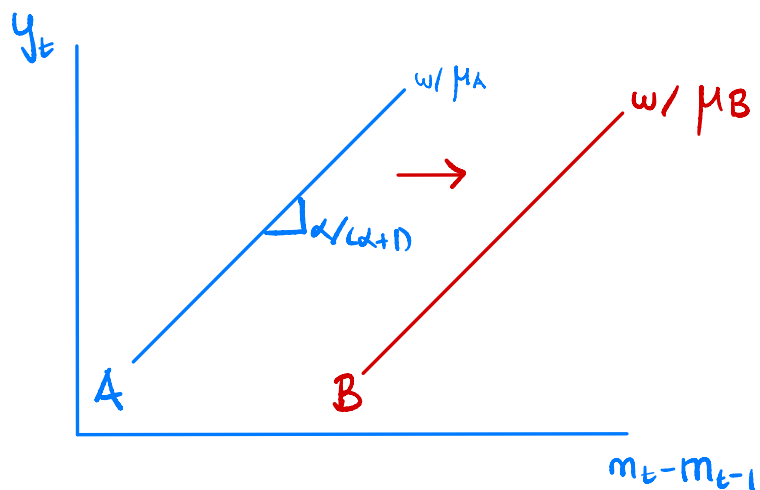
$$E_{t-1} m_t = \mu_A + m_{t-1}$$

Subst. $E_{t-1} m_t = \mu_A + m_{t-1}$

$$\Rightarrow y_t = \frac{\alpha}{\alpha+1} [m_t - m_{t-1} - \mu_A]$$

obs. growth rate of money

(*) y depends growth rate of money 'wedge' relative to μ_A



New rule: $m_t = \mu_B + m_{t-1} + \epsilon_t$
 $(\mu_B > \mu_A)$

$\rightarrow E[m_t - m_{t-1}] = \mu_B$

Key: Original relationship is valid only under original rule (3) (w/ μ_A)

In reality w/ new rule we should re-solve the system:

New AS would be: $y_t = \frac{\alpha}{1+\alpha} (m_t - m_{t-1} - \mu_B)$

Summary:

- When evaluating policy, need to take into account the feedback with expectations.
- If policy maker changes the rule, expectation will change so the equilibrium condition for the economy will change too.

⇒ All of the above point to the “danger” of using ad hoc, reduced-form empirical relationships with no “micro-foundation”!

Results of the RE revolution:

- 1) More focus on structural modeling of the economy (micro-foundation based on first principles), rather than estimating reduced-form equations. Identification key!
- 2) Taking expectations seriously
Lucas Critique does NOT imply policy ineffectiveness, but that policy making should focus on developing on-going strategy and long-term rules, not one-time change

⇒ **Four Main Approaches to Aggregate Supply**

Unification Again Since the 1990's

“New Neoclassical Synthesis”

- RBC techniques + New Keynesian insights
- Dynamic Stochastic General Equilibrium Modeling
- Business Cycles may be caused by real shocks, but nominal rigidity leads to inefficiency, hence role for policy

allows for market frictions and a role for Policy (to undo/mitigate frictions)

→ NK DSGE

- Modified AS
- " " AD
- Policy Rule

Key Questions – Extra credit

- What are some “stylized facts” about the US Business cycle dynamics (up to around 2000)?
- What is the Natural Rate Hypothesis?
- What does money neutrality mean?
- What is the Neoclassical Synthesis? And what were its main problems?

Part 3: Rational Expectations

A. Outline:

- Recap/Continuation of Rational Expectations

B. Housekeeping

- HW0 due on 1/5
- HW1 due on 2/19 (you'll be ready to start by today)

[Detour] [This is a good point for checking the Technical Slides #1]

[End of detour] Recap./Cont. of Rational Expectations in Macro:

Brief History of Modern Macro

- The Neoclassical Synthesis
- The Breakdown of the Consensus
- Is the Long-Run Phillips Curve Really Vertical?
- Rational Expectations Revolution
- Main Approaches to modeling Aggregate Supply since then
- The New Neoclassical Synthesis

IS-LM → AD-AS

Stagflation of 70's

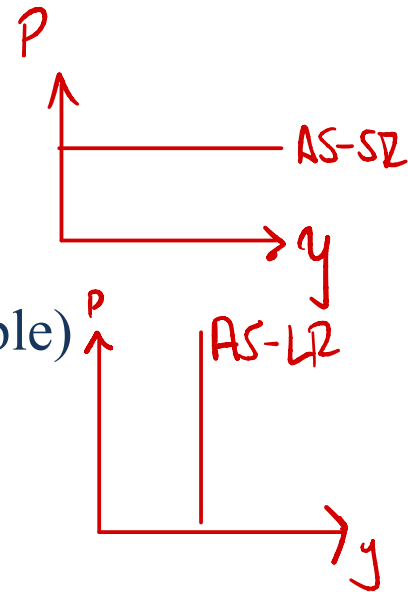
→ Reformulate a modern version
of AS-AD model
w/ Rational Exp.
Intertemporal Decisions

Summary from before:

Neoclassical Synthesis: Consensus in the 1960's

1. **Aggregate Demand**: comes from IS-LM framework
2. **Aggregate Supply**:
 - a) Short-run: Keynesian Nominal rigidity (sticky prices/wages)
 - b) Long-run: Natural Rate Hypothesis (prices/wages fully flexible)
3. **Phillips curve**: price adjustment from SR to LR

$$\pi = \alpha (Y - Y^N) = -\beta(u - u^N)$$



Phillips curve:

$$\pi_t = \alpha (Y_t - Y^N) = -\beta(u_t - u^N)$$

- Data in the 50's-60s, support above relationship
- Negative relationship implies a tradeoff between π and u and scope for policy actions

Breakdown of the Consensus due to:

- Friedman-Phelps' critique + Poor empirical support (in 70's)
- Lucas' Rational Expectation revolution

$$\pi_t = a - b u_t + \gamma E_{t-1} \pi_t$$

Lucas Critique ('73) and the Rational Expectation Revolution

- Nobel Prize 1995
- Much of what economists were doing and the policy conclusions were **WRONG**
 - Using a model with fixed coefficients estimated from reduced-form equations and historical data to evaluate the effects of new policy would give misleading results because **expectations need to be endogenous**
 - i.e. Changes in policy will affect expectations
- True whenever expectations are forward-looking (need not be rational)

Three Methodological Tenets for Rational Expectations²

- Results widely applicable; below look at examples regarding PC

1. Partial Equilibrium.

Agents form expectations appropriately given the stochastic process generating the variables of interest. Expectations cannot be specified without first specifying the underlying stochastic process.

How to measure $E_{t-1} \pi_t$?

² N.G. Mankiw

- Given the stochastic process, $E_{t-1}\pi_t = \rho\pi_{t-1}$ under Rational Expectation

- Plug this into (*), we see that

- $\pi_t = a - bu_t + \gamma\rho\pi_{t-1}$

↳ Need to take into account
Feedback of expectations into model

- What does this say about finding γ by looking at the historical relationship b/w inflation and unemployment?

[Further look into the implications of the stochastic process and the expectation operator]

- **Note:** Given $\pi_t = \rho\pi_{t-1} + \varepsilon_t$ where ε_t is a “white noise” process:
- ε_t is a time series process that’s independently and identically distributed (iid), with zero mean $E(\varepsilon_t) = 0$, a constant variance $E(\varepsilon_t^2) = \sigma^2$, and $E(\varepsilon_t \varepsilon_s) = 0$

Applying the Expectation operator E_{t-1} to π_t (taking its expectation at time t-1, implying based on information we have at time t-1):

$$\begin{aligned} E_{t-1}\pi_t &= E_{t-1}[\pi_t] = E[\pi_t|I_{t-1}] && \text{different notations for the same thing} \\ &= E_{t-1}[\rho\pi_{t-1} + \varepsilon_t] && \text{substituting in the definition of } \pi_t \\ &= E_{t-1}[\rho\pi_{t-1}] + E_{t-1}[\varepsilon_t] && \text{“operation” is linear (can do it term by term)} \\ &= \rho\pi_{t-1} + 0 && \text{what you expect at any time given info at that time = the info itself} \end{aligned}$$

Our (*) equation, the expectation-augmented Phillips Curve is then:

$$\begin{aligned}\pi_t &= a - bu_t + \gamma E_{t-1}\pi_t \\ &= a - bu_t + \gamma\rho\pi_{t-1}\end{aligned}$$

What does this imply?

- 1) the relation b/w inflation and unemployment depends on **lagged inflation** (π_{t-1}) too
- 2) Without specifying ρ first, we cannot determine γ

2. General Equilibrium.

The stochastic process of any variable generally depends on the entire model. To solve for expectations, one must assume that agents know and solve correctly the model of the economy.

- In our case, it means endogenizing π_t : have it determined from within the system
- We will adopt a variant of the Phillips curve (*) instead too.

Consider an economy described by a simple General Equilibrium (GE) model (note, all variables are in logs, allowing us to use linear equations):

$$1) m_t - p_t = y_t \quad (\text{AD})$$

$$2) y_t = \alpha(p_t - E_{t-1}p_t) \quad (\text{AS})$$

[Clarifying note: Relation btw Phillips Curve we saw before and (2) in last slide]

Note: The AS in the previous page is another way to express the expectation-augmented Phillips curve (*). From either of these expressions (again, equivalent):

$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$

$$\pi_t = -\beta(u_t - u^N) + \gamma E_{t-1}\pi_t$$

Since output y_t is inversely related to unemployment u_t :

$$\pi_t = cy_t + \gamma E_{t-1}\pi_t$$

Since inflation is the difference in prices: $\pi_t = p_t - p_{t-1}$

$$p_t - p_{t-1} = cy_t + \gamma E_{t-1}[p_t - p_{t-1}]$$

Re-arrange and note:

$$E_{t-1}[p_{t-1}] = p_{t-1}$$

$$y_t = \alpha(p_t - E_{t-1}p_t)$$

To Solve for Y from the system of AD and AS:

Fill:

Consider (1):

take E_{t-1} :

Take E_{t-1} of (2)

$$\Rightarrow E_{t-1}[p_t] = E_{t-1}[m_t]$$

$$\Rightarrow E_{t-1}[y_t] = 0$$

⋮

What does this result mean?

↳ Solve for y :

$$y_t = \frac{\alpha}{1 + \alpha} \underline{[m_t - E_{t-1}m_t]} \quad (*) \quad AS$$

- Output depends on “surprised” or unexpected money

3. Policy Evaluation

- Policy rules are part of the equations in most good macro models.
- A change in a policy rule alters the stochastic processes generating many variables => it also changes the way people form expectations.
- Any policy evaluation that fails to take account of this feedback is flawed.

Add a rule:

$$m_t = \mu_A + m_{t-1} + \varepsilon_t$$

$$\Rightarrow E_{t-1}[m_t] = m_{t-1} + \mu_A$$

μ_A : Indicates expected money growth

$$\begin{aligned} E_{t-1}[m_t - m_{t-1}] \\ = E_{t-1}[m_t] - m_{t-1} = \mu_A \end{aligned}$$

Key Questions (Extra Credit, due next session)

- What is the Lucas Critique?
- Intuitively, how do the three perspectives of rational expectations differ (and/or improve, complement each other)?
- How is Lucas' rational expectation captured within economic models?

Part 4: Approaches to model the Aggregate Supply

A. Reminder:

- HW1 due on by 2/19

B. Brief History of Modern Macro

- Rational Expectations Revolution (cont.)
- **Main Approaches to modeling Aggregate Supply since then**
- **The New Neoclassical Synthesis**

Summary from before:

Expectation-Augmented Phillips Curve:

$$\pi_t = -\beta(u_t - u^N) + \gamma E_{t-1}\pi_t$$

$$\pi_t = a - bu_t + \gamma E_{t-1}\pi_t$$

$$y_t = \alpha(p_t - E_{t-1}p_t)$$

Lucas Critique ('73) and the Rational Expectation Revolution

- Policy evaluation must consider how change in policy will affect (forward-looking) expectations
- “Rational”: model-consistent

To form “model-consistent” rational expectations:

1. Partial Equilibrium: specify the **stochastic process** generating the variables of interest

e.g. Given $\pi_t = \rho_1\pi_{t-1} + \rho_2\pi_{t-2} + \varepsilon_t$ where ε_t is a white noise process

- This is called an autoregressive process of degree 2, or AR(2), with the two ρ 's (“rho”) as the AR coefficients (constants)
- $E_{t-1}\pi_t = ?$

$$\begin{aligned} E_{t-1}\pi_t &= E_{t-1}[\rho_1\pi_{t-1} + \rho_2\pi_{t-2} + \varepsilon_t] \\ &= E_{t-1}[\rho_1\pi_{t-1}] + E_{t-1}[\rho_2\pi_{t-2}] + E_{t-1}[\varepsilon_t] \\ &= \rho_1\pi_{t-1} + \rho_2\pi_{t-2} \end{aligned}$$

2. General Equilibrium.

- The stochastic process of a variable generally depends on the entire model
- To solve for expectations, one must assume that agents know and solve correctly the model of the economy

e.g. (note, variables are in logs; see next page)

$$1) m_t - p_t = y_t \quad (\text{AD})$$

$$2) y_t = \alpha(p_t - E_{t-1}p_t) \quad (\text{AS})$$

$$y_t = \frac{\alpha}{1 + \alpha} [m_t - E_{t-1}m_t]$$

=> Output depends on “surprised” or unexpected money

[Clarification-Why we use logs or similar approximations]

A note about **linearized equations**:

- We will be working with “linearized” equations frequently, to make the models easier to solve.
- In order to express general economic equations in linear forms, we often take the (natural) logs of the variables. For example, the quantity theory of money, $M_t V_t = P_t Y_t$ can be expressed as $m_t + v_t = p_t + y_t$ where each of the lower-case variables is the log of the capitalized variables.
- We will use variations of $m_t + v_t = p_t + y_t$ to represent **aggregate demand** for a while.

3. Policy Evaluation (where does m_t come from?)

- The rules governing policy are among the equations in any complete model of the economy
- Because a change in a policy rule alters the stochastic processes generating many variables, it also changes people's expectations
- Hence, when evaluating alternative policy rules, one must take account of this feedback between policy rules and how expectations are formed.
- Any policy evaluation that fails to take account of this feedback is flawed and useless (Lucas Critique)

(3) Consider a simple monetary policy rule: $m_t = \mu_A + m_{t-1} + \varepsilon_t$
where μ_A (“mu”_A) is a constant and ε_t is a **white noise** process gain

Note: since these variables are in logs, $\mu_A = E(m_t - m_{t-1})$
represent the **expected growth rate of money** or the **trend money growth rate** (Remember: log (small) differences = % change or growth rate.)

Combining (1)-(3) to solve the GE model:

$$E_{t-1}m_t = \mu_A + m_{t-1}$$
$$\Rightarrow y_t = \frac{\alpha}{1+\alpha} [m_t - m_{t-1} - \mu_A]$$

- This tells us that in general equilibrium, the level of output at each time depends on both money growth from the previous period ($m_t - m_{t-1}$), but also on the general trend growth rate set by monetary policy (μ_A)

 μ_B

What is the policy implication from this rational expectation general equilibrium model?

- Output level is positively correlated with money growth from period-to-period. However, this relationship does NOT imply **policy tradeoff** because the line above is only valid under the policy rule (3) with μ_A
- If policymaker increases money growth rate from point A to point B (e.g. by changing money growth from μ_A to μ_B), (3) would no longer be the correct rule. The rule would have trend money growth μ_B instead
- People forming rational expectations would adjust as well based on the new rule. Resolving the system, we see that the curve shifts to the right (to reflect the new output-money growth relationship)

Results of the RE revolution:

- 3) More focus on structural modeling of the economy: **micro-foundation based on first principles**, rather than estimating reduced-form equations. **Identification: correlation does not equal causality**
- 4) Taking expectations seriously
Lucas Critique **does NOT imply policy ineffectiveness**, but that policy making should focus on developing on-going strategy and long-term rules, not one-time change

Next: Four Main Approaches to Aggregate Supply

Markets are frictionless \Rightarrow Policy Intervention is unnecessary

		Do Markets Clear? <small>(instantaneously)</small>	
		Yes	No
Is Money Neutral? <small>(in medium, Long run)</small>	Yes	1. Classical/RBC Kydland & Prescott Minnesota	3. Real Rigidity e.g. Efficiency wage theory Akerlof, Yellen
	No	2. Imperfect Information Friedman Lucas '77	4. Nominal Rigidity Nominal contracts, menu costs: Fischer, Taylor, Calvo ...

There are market frictions such as Rigid Prices that create inefficiencies in private markets that lead to suboptimal outcomes \Rightarrow Role for Government Intervention.

Unification Again Since the 1990's

“New Neoclassical Synthesis”

} → approach 4: Post Keynesians
RBC toolkit + Market Frictions

- Real Business Cycle tools (Romer Ch.5) + New Keynesian (Ch. 6) ideas
- Dynamic Stochastic General Equilibrium Modeling
- Business cycle may be caused by real shocks, **but nominal rigidity leads to inefficiency, hence role for policy**

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A Primer"