Problem set # 0

Due date: January 29

This homework is assigned with the objective of making you review some mathematical tools that will be very useful in the rest of the course. It is not macroeconomics yet, but if this material is clear you'll be able to focus on the economic insights of the models to follow rather than on the technical details.

You'll get full credits for this homework as long as you make a complete submission.

If you find very difficult to solve three or more of these problems, you should review calculus and come to my office hours for further explanations.

1. Find $\frac{\partial f}{\partial x}$ for $f(x) = \frac{x^{1-\sigma}}{1-\sigma}$ where $\sigma > 0$ is a constant.
2. Find $\frac{\partial f}{\partial x}$ when $f(x, y) = x^3 y \ln x$, where $\ln(\cdot)$ is the natural logarithm.
3. If $f(x, \beta) = (\frac{x}{\beta^2})e^{-x/\beta}$, find $\ln f(x, \beta)$. Use this to find $\frac{\partial}{\partial \beta} \ln f(x, \beta)$
4. For $0 < \beta < 1$, show that $\sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$
5. **[Basic constrained optimization - application from microeconomics]**
   Consider this optimization problem:[1]

$$\max_{x,y} U(x, y) = (x + 1)(y + 1) = xy + x + y + 1$$
$$\text{subject to } x + 2y = 30$$

   Find the optimal levels of x, y that solve the optimization problem above.

   For this, set the associated lagrangian, find the First Order Conditions and solve for x and y.

6. **[Dynamic optimization first order condition]**

   Consider the function $L = \sum_{s=0}^{\infty} \beta^{t+s} \left[\sqrt{x_{t+s}} + 35 + \lambda_{t+s}(100 - \frac{1}{2}x_{t+s} + \frac{4}{5}x_{t+s-1})\right]$

   Find the derivative of this function with respect to x_t.

[1] The actual problem has as budget constraint $x + 2y \leq 30$, and non-negativity constraints for x and y. Based on economic theory we know we can solve the simpler problem above where the first constraint binds and the quantities consumed of each commodity is larger than zero.