Econ 5322
Granados

Calvo Staggered Price Setting and the New Keynesian Phillips Curve

Consider a representative monopolistically competitive firm with its optimal price p_t^*:

$$p_t^* = p_t + \phi y_t,$$

where ϕ is the degree of real rigidity (or strategic complementarity in the price setting), p_t is the log CPI, and y_t is the log of output gap. Note that this kind of optimal price is a feature of most models of monopolistic competition (e.g., Blanchard-Kiyotaki), and it is the same expression we used in the Taylor model discussion: $p_t^* = \theta m_t + (1-\theta)p_t$, using $m_t - p_t = y_t$

Consider an arbitrary profit function $\Pi(x_t)$ for firm with current price x_t. Its second-order Taylor approximation around p_t^* is:

$$\Pi(x_t) - \Pi(p_t^*) = \frac{\Pi''(p_t^*)}{2}(x_t - p_t^*)^2$$

Note that the above takes into account the optimality condition of the firm, the first-order condition (F.O.C.): $\Pi'(p_t^*) = 0$. The S.O.C. implies $\Pi''(p_t^*) < 0$. We can write the quadratic instantaneous loss function of the monopolistic competitors as:

$$L_t = \frac{K}{2}(p_t^* - x_t)^2,$$

where K is the absolute value of the second derivative of the profit function. This function captures the profit loss to the firm when its current price x_t deviates from the optimal price p_t^*.

The Calvo assumption is that a representative firm gets to reset its price in any given period with probability $(1-\theta)$. (Note; a higher θ implies higher nominal ridigity.) The objective of the firm resetting its price at period t is to minimize the expected profit loss over the (expected) duration of being stuck with this price, which is given by

$$\min_{x_t} \sum_{j=0}^{\infty} (1-\theta)\theta^j \beta^j \frac{K}{2}(E_t p_{t+j}^* - x_t)^2$$

where β is the discount factor. Here $(1-\theta)\theta^j$ is the probability that the firm will not reset its price until period $t+j+1$. Therefore, the objective is the unconditional expectation of the (discounted) loss of the firm from setting price x_t in period t while each term in the sum is the conditional expectation of the (discounted) loss in period $t+j$ given that the firm would not yet get to change its price by period $t+j$.

The F.O.C. for the firms optimization is:

$$K(1-\theta)\sum_{j=0}^{\infty} \theta^j \beta^j (E_t p_{t+j}^* - x_t) = 0$$

Expressing out and solving for x_t gives us the price firm should set:

$$x_t = (1-\theta\beta)\sum_{j=0}^{\infty}(\theta\beta)^j E_t p^*_{t+j} \tag{x}$$

In words, x_t is the average optimal price for the firm across all future periods, weighted by the probabilities that this price remains effective in these periods.

We can rewrite the expression for x_t in a difference equation form (please confirm this for yourself):

$$x_t = (1-\beta\theta)p^*_t + \beta\theta E_t x_{t+1}$$

Then substitute in the expression for p^*_t:

$$x_t = \beta\theta E_t x_{t+1} + (1-\beta\theta)p_t + (1-\beta\theta)\phi y_t$$

Define a new variable $z_t = x_t - p_t$ and rewrite the above:

$$z_t - \beta\theta E_t z_{t+1} = \beta\theta E_t \pi_{t+1} + (1-\beta\theta)\phi y_t \tag{*}$$

where inflation $\pi_{t+1} \equiv p_{t+1} - p_t$.

The final step is to note that the aggregate price level in the economy is the average price across all firms. Since fraction $(1-\theta)\theta^j$ of the firms have set their price exactly $t-j$ periods ago,

$$p_t = (1-\theta)\sum_{j=0}^{\infty}\theta^j x_{t-j}$$

We can rewrite the last expression as

$$\begin{aligned} p_t &= (1-\theta)x_t + \theta p_{t-1} \Longrightarrow \qquad (+) \\ \theta\pi_t &= (1-\theta)(x_t - p_t) = (1-\theta)z_t \end{aligned}$$

Rewrite it in difference form:

$$(1-\theta)(z_t - \beta\theta E_t z_{t+1}) = \theta(\pi_t - \beta\theta E_t \pi_{t+1})$$

and substitute into (*) above, we can get rid of z_t and obtain the "classical" Calvo Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda y_t,$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\phi$.

Remark 2: λ determines the degree of output-inflation trade-off – the curvature of the Phillips Curve (or Aggregate Supply). The smaller λ the larger the real effect of inflation on output: λ decreases as ϕ decreases (i.e., more real rigidity) and as θ increases (i.e., more nominal rigidity).

Remark: Note that another measure of the degree of real rigidity – $K \equiv -\Pi''(p^*_t)$– does not affect the form of the Phillips Curve. However, it is important because the smaller K the less one needs menu costs (broadly defined) to motivate this kind of behavior.