Problem set # 5

Due date: April 27

Please show your work carefully in answer the following questions. As mentioned in class, you are encouraged to work in groups but must write your own answers.

1. **(Romer 5 ed. 8.6 - Excess Smoothness)** Suppose that $C_t = \frac{r}{1+r}\left\{A_t + \sum_{s=0}^{\infty}\frac{E_t[Y_{t+s}]}{(1+r)^s}\right\}$ and that $A_{t+1} = (1+r)(A_t + Y_t - C_t)$, where A_t denotes the wealth at time t, C_t the consumption, Y_t the income (also at time t for both), and r the interest rate which we assume constant over time. Notice that this is the same formula we discussed in the lectures when we mentioned the consumption is given by a fixed-average- fraction of the expected lifetime resources.

   (a) Show that these assumptions imply that $E_t[C_{t+1}] = C_t$ (and thus consumption follows a random walk) and that $\sum_{s=0}^{\infty}\frac{E_t[C_{t+s}]}{(1+r)^s} = A_t + \sum_{s=0}^{\infty}\frac{E_t[Y_{t+s}]}{(1+r)^s}$.

   This is important because it helps you understand why the random walk property is crucial if we want to think of consumption as a fixed and predetermined fraction of the future expected resources.

   [Hint: to obtain the formula for C_t we actually used the random walk assumption, then here you just have to verify the formula is consistent with the assumption made. A good way to go about it is to find an expression for $E_t[C_{t+1}]$ and compare it to that of C_t, when doing so don't forget to use the formula for A_{t+1} when setting C_{t+1}; for the second part, realize that $\frac{r}{1+r} = [\sum_{s=0}^{\infty}\frac{1}{(1+r)^s}]^{-1}$ and that the random walk result allows you to take in an out of the summation the consumption.]

   (b) Suppose that $\Delta Y_t = \phi\Delta Y_{t-1} + u_t$, where u is white noise. Suppose that Y_t exceeds $E_{t-1}[Y_t]$ by 1 unit (that is, suppose $u_t = 1$). By how much does consumption increase?

   Here, we are just thinking in term of impulse responses in the income growth. We are particularly interested in knowing what is the response of consumption after the income turned out to be different from what expected. Why would consumption act out after the shock? you will see here that a shock today, will change the stream of expected income for some periods ahead and that should be accounted when revising our consumption.

   [Hint: set an expression for $E_{t-1}[C_t]$ and obtain $C_t - E_{t-1}[C_t]$, do the same with the present value of the expected lifetime income (the Y part in the expression for consumption), this part will be affected by the shock, particularly due to its persistence over time, i.e., ϕ.]

   (c) For the case of $\phi > 0$, which has a larger variance, the innovation in income, u_t, or the innovation in consumption, $C_t - E_{t-1}[C_t]$? Do consumers use saving and borrowing to smooth the path of consumption relative to income in this model? Explain.

   We are interested in knowing how the volatility of the shock translates into the volatility of the consumption adjustment. During lecture and in the tech sessions, we discussed how the endogenous variables end up absorbing the time series (mean, volatility, etc.) properties of the shocks. This is no exception. Of particular interest is to know whether the consumption will over-react or under-react to the shock which is something we can know by determining whether it is more volatile or not.

[Hint: Given it is a white noise, the variance of u_t is just the parameter σ^2. Compare that to the variance of the result you got in (b)]

2. **(Comparison of models solutions)** Here you are going to run the Dynare files associated to two known models in the literature and compare their results.

   (a) Consider the file "Gali_2008_chapter_2.mod", it contains the code for setting up and solving the RBC model of the Chapter 2 in Jordi Gali's 2008 book[1]. This corresponds to one of the types of models we mentioned when discussing the approaches to model the aggregate supply (the one without frictions and instantaneous market clearing). Run the file and report its output and impulse responses.

   (b) Now, consider the file "Gali_2008_chapter_3.mod". It contains the code for setting up and solving the New-Keynesian model of the Chapter 3 in the same book. This is also one of the types of models we covered, where we add nominal rigidities to the frictionless RBC framework. Run the file and report its output and impulse responses.

   (c) Now, let's study the implications on money neutrality. Before checking the IRF's, what are you expecting?. Should money be neutral in either of the models? why or why not?

   (d) Now take a look at the IRF's of each model. If the response of a variable to a certain shock is zero at all times, Dynare does not report an IRF (after all, there's no "impulse" to show in that case). That is why for either shock you may have a different number of graphs showing up. How can you relate that (the impulses showing up) to the money neutrality? [hint: focus in the case of a monetary shock, i.e., the shocks to "eps_m" for the first model and "eps_nu" for the second model. If it helps, also compare the responses in those cases with that of "real" technological shocks "eps_A" or "eps_a"]

[1] Galí, Jordi, (2008)."Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications" Economics Books, Princeton University Press, first edition.