Problem Set # 3

Answer Key

1. **(Romer, 5th ed. 6.16) Observational Equivalence.**[1] (Sargent, 1976). Suppose that the money supply is determined by $m_t = cz_{t-1} + e_t$, where c is a coefficient, and z an economic variable and e_t is an i.i.d. disturbance uncorrelated with z_{t-1}. e_t is unpredictable and unobservable. Thus the expected component of m_t is cz_{t-1}, and the unexpected component is e_t. In setting the money supply, the Federal Reserve responds only to variables that matter for real activity; that is, the variables in z directly affect y.

   Now consider the following two models: (i) Only unexpected money matters, so $y_t = az_{t-1} + be_t + v_t$; (ii) all money matters, so $y_t = \alpha z_{t-1} + \beta m_t + v_t$. In each specification, the disturbance is i.i.d. and uncorrelated with z_{t-1} and e_t.

   (a) Is it possible to distinguish between these two theories? That is, given a candidate set of parameter values under, say, model (i), are there parameter values under model (ii) that have the same predictions? Explain.

(Ans) The models (i) and (ii) are indistinguishable in this case (when the monetary rule is $m_t = cz_{t-1} + e_t$).

To see that, replace the monetary rule in the supply equation in model (ii):

$$\begin{aligned} y_t &= \alpha z_{t-1} + \beta(cz_{t-1} + e_t) + v_t \\ &= (\alpha + \beta c)z_{t-1} + \beta e_t + v_t \end{aligned}$$

Notice that if $a = (\alpha + \beta c)$ and $b = \beta$ the structure of models (i) and (ii) is identical (i.e. they are undistinguishable).

Intuitively, this implies that from the perspective of what is observed (y_t and m_t for example), the public, even if rational cannot tell whether the policymaker is setting the its policy according to a framework where "all money matters" or where "unexpected money matters".

   (b) Suppose that the Federal Reserve also responds to some variables that do not directly affect output; that is, suppose $m_t = cz_{t-1} + \gamma w_{t-1} + e_t$ and that models (i) and (ii) are as before (with their disturbances now uncorrelated with w_{t-1} as well as with z_{t-1} and e_t). In this case, is it possible to distinguish between the two theories? Explain.

[1] The original exercise in the book treats a, α, c and γ as a vector of coefficients, and z and w as a vectors of economic variables. Feel free to solve this simpler version of the original one. The idea behind is the same.

(Ans) Now $m_t = cz_{t-1} + \gamma w_{t-1} + e_t$ (w_{t-1} does not affect output)

With this new structure the model (ii) becomes:

$$\begin{aligned} y_t &= \alpha z_{t-1} + \beta(cz_{t-1} + \gamma w_{t-1} + e_t) + v_t \\ &= (\alpha + \beta c)z_{t-1} + \beta\gamma w_{t-1} + \beta e_t + v_t \end{aligned}$$

In this case it is possible to distinguish the models as there is no combination of parameters such that (ii) is identical to (i) (as long as β and b are not zero). Then, we have that as long as $\beta\gamma \neq 0$ we are able to distinguish cases in which all money matters (w_{t-1} matters too) from the case where only the unexpected component matters (e_t).

2. **(Matlab question: more on autorregressive processes)** Consider the following autorregressive process of the second order, AR(2):

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \epsilon_t$$

Where $\epsilon_t \overset{i.i.d.}{\sim} N(0,1)$. Suppose that the process begins at $t = 0$, so all values before that are equal to zero. At $t = 0$, the system is shocked with $\epsilon_0 = 1$, thereafter the shocks are all zero ($\epsilon_1 = 0, \epsilon_2 = 0$ and so on).

With everything else constant but the shock, we call the resulting sequence of y_t's an impulse response to a shock in ϵ.

(a) Show that $\{y_0, y_1, y_2\} = \{1, \alpha_1, \alpha_1^2 + \alpha_2\}$. Also, obtain y_3.

(Ans) $y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \epsilon_t$, also $\epsilon_t = 0$ for all t but $t = 0$ where $\epsilon_0 = 1$, and every $y_t = 0$ for $t < 0$.

$$y_0 = \alpha_1 y_{-1} + \alpha_2 y_{-2} + \epsilon_0$$

Substitute: $y_{-1} = y_{-2} = 0$, $\epsilon_0 = 1$,

$$y_0 = 1$$

$$y_1 = \alpha_1 y_0 + \alpha_2 y_{-1} + \epsilon_1$$

Substitute: $y_0 = 1, y_{-1} = 0$, $\epsilon_1 = 0$

$$y_1 = \alpha_1$$

$$y_2 = \alpha_1 y_1 + \alpha_2 y_0 + \epsilon_2$$

Substitute: $y_1 = \alpha_1, y_0 = 1, \epsilon_2 = 0$

$$y_2 = \alpha_1^2 + \alpha_2$$

$$y_3 = \alpha_1 y_2 + \alpha_2 y_1 + \epsilon_3$$

Substitute: $y_2 = \alpha_1^2 + \alpha_2$, $y_1 = \alpha_1$, $\epsilon_3 = 0$

$$y_3 = \alpha_1(\alpha_1^2 + \alpha_2) + \alpha_2\alpha_1 = \alpha_1^3 + 2\alpha_1\alpha_2$$

(b) Now assume $\alpha_1 = 0.9$, $\alpha_2 = -0.1$. Using Matlab, obtain and plot the impulse responses for 24 periods after the shock, i.e. get $\{y_0, y_1, y_3, \cdots, y_{24}\}$. For your submission report the plot only (not the values).

[*Hint:* this would be a pain to do by hand, but in Matlab, it can be implemented very easily with a for loop.]

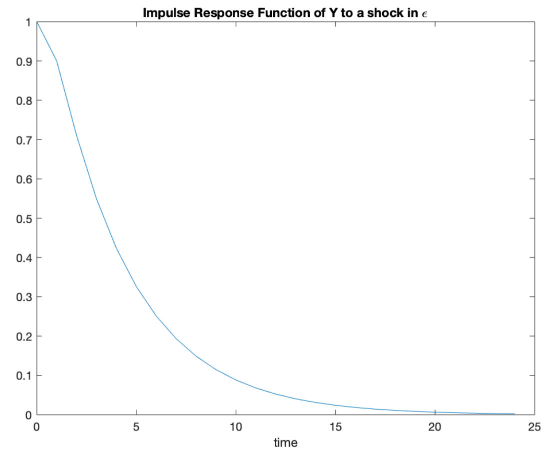
Can you see how the process returns to its expected value after some time (once the effect of the shock dies out)? That is the typical behaviour we expect to see in an impulse response describing a standard "stationary" process. In the rest of the exercise you will see a process where this does not happen.

(Ans) $\alpha_1 = 0.9$, $\alpha_2 = -0.1$

Code used:

```
% Solution to (b) ((d) with alpha1=1 and alpha2 = 0)

alpha1 = 0.9;
alpha2 = -0.1;

y = zeros(25,1); %first we create a matrix to fill with our results

eps0 = 1;
epsOther = 0;

y(1) = alpha1*0 + alpha2*0 + eps0; %the first position is t=0
y(2) = alpha1*y(1) + alpha2*0 + epsOther;

for i = 3:length(y)
y(i) = alpha1*y(i-1) + alpha2*y(i-2) + epsOther;
end

plot(0:(length(y)-1),y) %x part in plot represents time, then starts at zero
title("Impulse Response Function of Y to a shock in \epsilon"); xlabel("time")
```

The plot is:



Interpretation: the process departs from its mean (zero) due to the shock. Over time the effect dies out and the process reverts back (hence the label "mean reverting processes").

(c) Now let's assume $\alpha_1 = 1$, $\alpha_2 = 0$. The resulting process (no longer AR(2)), is called a Random Walk process (a special type of AR(1) where the autorregressive coefficient is equal to one). It is called "random" because the effect of a shock does not dissapear over time and then, after a shock the process does not revert to its mean value, which ultimately implies that the shocks (which are random and stochastic by nature) will dictate where the process goes.

I want you to see this yourself, first algebraically, and then with a program.

Find expressions for $\{y_0, y_1, y_2, y_3, y_4\}$

Does the effect of the shock fade out as time goes on?

(Ans) $\alpha_1 = 1$, $\alpha_2 = 0$

Algebraically we can verify:

$$y_0 = \alpha_1 y_{-1} + \epsilon_0 = (1)(0) + 1 = 1$$

$$y_1 = \alpha_1 y_0 = (1)(1) = 1$$

$$y_2 = \alpha_1 y_1 = (1)(1) = 1$$

$$y_3 = \alpha_1 y_3 = (1)(1) = 1$$

$$y_4 = \alpha_1 y_4 = (1)(1) = 1$$

(d) Now, again with $\alpha_1 = 1$, $\alpha_2 = 0$, plot the impulse response for the 24 periods after the shock, i.e., $\{y_0, y_1, y_3, \cdots, y_{24}\}$.

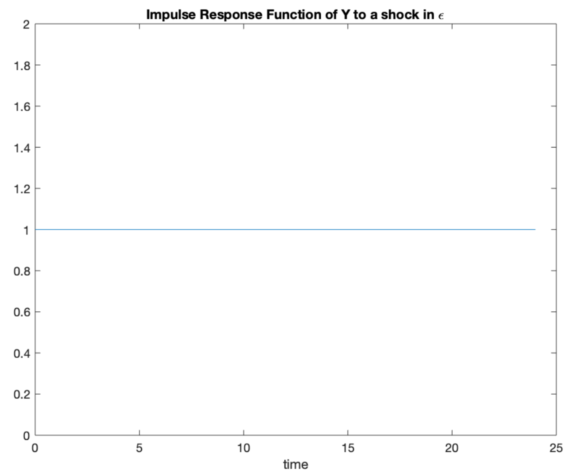
Does the effect of the shock fade out as time goes on even more periods?

(Ans) $\alpha_1 = 1$, $\alpha_2 = 0$

The code is exactly the same as above (in (b)), except for the lines:

```
alpha1 = 1;
alpha2 = 0;
```

The resulting plot is:



We can see that the shock makes y depart from its mean, and due Random Walk feature ($\alpha_1 = 1$) the effect of the shock is permanent, hence the process never reverts back to its expected value.

3. Explain whether the following statement is *true, false or uncertain*:

   (a) **(Lucas Island Model)** According to the Lucas imperfect information model, a 5% drop in the money supply will have a larger effect on output in an economy where monetary conditions are stable, compared to an economy with volatile monetary conditions.

(Ans) True: In the Lucas island model each producer decision is based in the following reaction function,

$$y_j = \gamma(p_j - E(P|I_j)) = \gamma\left(\frac{\tau^2}{\tau^2 + \sigma^2}\right)p_j \tag{1}$$

where p_j is j- island's (producer) product price and τ^2 and σ^2 are the volatility of the idiosyncratic and common shock, respectively. Since σ^2 is common to every island it represents the stability of the economy as a whole (the lower the more stable).

In this scheme $\gamma\left(\frac{\tau^2}{\tau^2+\sigma^2}\right)$ is the response coefficient of production upon changes in the price, which at the same time is reacting to monetary conditions. In this context, the statement is TRUE since as σ^2 approaches zero ($\sigma^2 \to 0$ or conditions are more stable) the coefficient is higher and so is the adjustment of the ouput (y_j).

Here, I am just mentioning the invididual response of a firm j. You should also think how this eventually relates to the Aggregate Supply Fuction (and curve) after we account for the decision making made by all of the firms. The result is that the same reasoning applies in the aggregate.

The question specifically wants you to understand the Lucas supply curve. One way to look at this question is to understand how the agent forms expectations. The second part of the question gives you situations: i) stable monetary conditions and ii) volatile monetary conditions. In either situation, the agent tries to deduce whether changes in their prices are due to sector specific shocks or due to general price increases. Firms want to change output in the former case. However, under volatile conditions, firms would think most changes in their prices are due to changes in general prices, and therefore would be less reponsive. Hence, AS curve is expected to be steeper. Hence, the effect of money on output is larger under stable conditions, when less of the money shock is anticipated.

(b) **Fischer Model** According to the Fischer wage contracting model, a change in money supply affects real economic activity only if it is not fully expected at the time contracts are signed.

(Ans) False. A deterministic (i.e. fully expected) monetary policy rule can still have stabilizing effects. That is, even if the level of output is only affected by policy surprises, policymakers can still offset the demand shocks, given the informational advantage they have since part of the population cannot observe these shocks when wages are set.

Here what you need to be careful with, is in understanding that although the level is not affected, policy is still relevant and affects the output by lowering its volatility.