Problem set # 1

Due date: February 16

Answer the following questions. Show your work. As mentioned in class, you are encouraged to work in groups but must write your own answers.

1. **(Simple model, rational expectations)** Consider an economy that is described by the following conditions:

$$w_t = \mathbb{E}_{t-1} p_t \quad (1) \quad \text{[one period wage stickyness]}$$
$$y_t = -(w_t - p_t) \quad (2) \quad \text{[Aggregate Supply, AS]}$$
$$y_t + p_t = m_t + \tilde{v}_t \quad (3) \quad \text{[Aggregate Demand, AD]}$$
$$m_t = b(L) v_{t-1} \quad (4) \quad \text{[Deterministic Monetary Policy Rule]}$$

With $\tilde{v}_t = D(L) v_{t-1} + \eta_t$, $\eta_t \sim N(0, \sigma^2)$.

Where y_t: Output in period t, w_t: wages, p_t: price level, $\mathbb{E}_{t-1}[\cdot]$: expectation operator with information up to $t-1$, $\tilde{v}_t$: observed aggregate demand shock, η: stochastic or surprise component of the AD shock, v_t: deterministic component of the AD shock.

(Here $D(L), b((L)$ correspond to polynomials of coefficients with the L denoting a "lag" operator such that $L^k x_t = x_{t-k}$. For the solution of this problem you can abstract from these details and assume that $D(L), b(L)$ are coefficients).

   (a) Determine which variables are endogenous and which are exogenous.

   (b) Solve for the endogenous that is an expectation ($\mathbb{E}_{t-1} p_t$) as a function of the exogenous variables.

   (c) Solve for the remaining endogenous variables

2. **(Empirical exercise)** In this exercise you will update the macroeconomic time series plots from lecture 1 for the US economy[1]. Download the quarterly GDP data and another one of the following variables: total consumption, durable goods consumption, non-durable goods consumption, investment, employment.

   (a) Using the code "ps1q2_code" (change the corresponding lines so that it runs in your pc and be sure to include "hpfiler.m" in your working folder) plot the log of the level and the trend in one graph and the cycle in another graph. Do this for each of the two variables.

   (b) For the cycle of each variable compute the standard deviation, the correlation with the output, and the autocorrelation.

[1] the series can be found at the Federal Reserve Economic Data (https://fred.stlouisfed.org/). Keep in mind that the variables you are interested in are the real ones (not nominal).

(c) What can you interpret about these variables after updating? Do you think we are still going through a Great Moderation of the business cycles as in the early 2000's?

3. **(Autorregresive processes)** The following process, denoted AR(1), describes the behavior of y_t:

$$y_t = \alpha y_{t-1} + \epsilon_t \tag{1}$$

$$\epsilon_t \overset{i.i.d.}{\sim} N(0, \sigma^2) \tag{2}$$

With $|\alpha| < 1$. Notice the error term (ϵ_t) is a white noise process, exogenous and uncorrelated with current and past realizations of y. Also, this AR(1) process is characterized by having well defined moments (expectation, variance, etc.) that do not depend on t (i.e. $E[y_t] = E[y_{t-1}]$ and the same for higher order moments).

(a) Using the expectation operator ($\mathbb{E}[\cdot]$) obtain the expected value of y_t

(b) Obtain the variance of y_t (Hint: you can find the variance of a random variable X as $V(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$).

(c) find the covariance of y_t with y_{t-1}. (Hint: in general $Cov(AB) = \mathbb{E}[AB] - \mathbb{E}[A]\mathbb{E}[B]$)