Econ 5322 - MS Macroeconomics Spring 2025
Granados

Problem set # 2

Due date: March 3

Please show your work carefully in answer the following questions. As mentioned in class, you are encouraged to work in groups but must write your own answers.

1. **(RBC model with elastic labor supply)** Consider the RBC model studied in class. Assume that labor supply is variable: Agents choose optimally how much labor to supply in each period. The period utility takes the form:

$$u(C_t, N_t) = \frac{\left[C_t^{\rho}(1-N_t)^{1-\rho}\right]^{1-\gamma}}{1-\gamma}$$

with $\gamma > 0$ and $0 < \rho < 1$ Utility is no longer separable between consumption and leisure (unless we assume $\gamma = 1$). All other assumptions are unchanged (relative to the model studied in class).

   (a) Write the first-order conditions that determine agents' optimal behavior. Explain these first-order conditions intuitively.

   [Hint: The marginal utility of consumption (or leisure) depends on leisure (or consumption) now. In addition to usual intepretations, consider this when comparing to the fixed-labor case.]

   (b) Assume $\gamma = 1$ for the rest of this problem. This implies that the utility function becomes $\rho \log C_t + (1-\rho)\log(1-N_t)$. Solve for the balanced growth path of the model and log-linearize the model around it using the technique described in the lecture.

   [Hint: with $\gamma = 1$ you are looking at a special case of the variable-labor model we studied in class. Rewrite the utility function as $\rho\left[\log C_t + (1-\rho)\log(1-N_t)/\rho\right]$ and note that maximizing this is the same as maximizing the original utility function. Compare this to the utility function for the variable-labor model in the slides and you should see that, if you set $\theta = (1-\rho)/\rho$, you have a special case of the slides, which you can follow to solve the problem.]

   (c) Use the "guess and check" procedure shown in class and the method of undetermined coefficients to solve the model

2. **(RBC model with government spending shocks)** Consider again the stochastic growth model, focus on the fixed-labor case, but now allow for government spending shocks as a source of fluctuations.

   The representative consumer maximizes:

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \frac{C_{t+i}^{1-\gamma}}{1-\gamma}$$

   where $0 < \beta < 1$ and $\gamma > 0$.

The law of motion of capital is:

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t - X_t \tag{*}$$

X denotes exogenous government spending, financed through lump-sum taxation.

Output is:

$$Y_t = A_t^{\alpha} K_t^{1-\alpha}, \quad \text{with } 0 < \alpha < 1$$

(a) Obtain the Euler Equation for capital accumulation. Explain the intuition.

(b) Solve for the balanced growth path (here it's useful to treat $\bar{X}_t/\bar{Y}_t$ as an exogenous variable). Assume $\bar{A}_{t+1}/\bar{A}_t = \bar{X}_{t+1}/\bar{X}_t = G$.

(c) Assume log-normality, so that $\log(\mathbb{E}_t X_{t+1}) \approx \mathbb{E}_t(\log X_{t+1}) + \frac{1}{2} Var_t(\log X_{t+1}))$ for any variable X, and homoskedasticity (variances and covariances are constant).

Log-linearize equation (*), the Euler Equation, and the expression for the gross return to capital accumulation around the steady state.

(d) Assume $a_t = 0$ for all t (i.e., there are no percentage deviations of technology from the steady state). Assume $x_t = \phi x_{t-1} + \varepsilon_t$, $\mathbb{E}_{t-1}\varepsilon_t = 0$. Show that the model reduces to:

$$k_{t+1} = \lambda_1 k_t + \lambda_4 x_t + (1 - \lambda_1 - \lambda_2 - \lambda_4) c_t$$
$$\mathbb{E}_t(c_{t+1} - c_t) = -\frac{\lambda_3}{\gamma} k_{t+1}$$
$$x_t = \phi x_{t-1} + \varepsilon_t$$

with $\lambda_1 \equiv \frac{1+r}{1+g}$, $\lambda_2 \equiv \frac{\alpha(r+\delta)}{(1-\alpha)(1+g)}$, $\lambda_3 \equiv \frac{\alpha(r+\delta)}{1+r}$, $\lambda_4 = -\frac{(r+\delta)\bar{X}_t/\bar{Y}_t}{(1-\alpha)(1+g)}$, and $\mathbb{E}_{t-1}\varepsilon_t = 0$.

(e) The solution for consumption and capital has the form:

$$c_t = \eta_{ck} k_t + \eta_{cx} x_t$$
$$k_{t+1} = \eta_{kk} k_t + \eta_{kx} x_t$$

What is the intuition for this solution? Briefly describe how these equations and $x_t = \phi x_{t-1} + \varepsilon_t$ can be used to trace the response of capital and consumption to a government spending shock.

3. Briefly: Describe the possible calibration approaches to evaluate the empirical performance of models.