Problem Set #2 – ECON 3311, Spring 2025 **[Answer Key]**

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1. **Suppose the production function of an economy is given by $Y = \bar{A}\ K^{1/4}L^{3/4}$**

   a. **Calculate GDP per capita (y) as a function of $\bar{A}$ and k?**

   To arrive at GDP per capita we divide the production function by L:

$$y = \frac{Y}{L} = \bar{A}\frac{K^{1/4}\,L^{3/4}}{L^{1/4}} = \bar{A}\ \frac{K^{1/4}}{L^{1/4}} = \bar{A}\ k^{1/4}$$

   b. **Does the production function display constant, decreasing, or increasing returns to scale?**

   With Cobb-Douglas production functions, returns to scale depends on the sum of the exponents on *K* and *L*. In this case, because they are 1/4 and 3/4, their sum is equal to 1 and the production function displays constant returns to scale.

2. **The following table depicts observed GDP per capita and predicted GDP per capita for several countries relative to the US:**

Production Model's Prediction for per Capita GDP (U.S. = 1)

	Observed per capita GDP	Predicted per capita output, $y_P = k^{1/5}$
Burundi	0.02	0.19
Brazil	0.29	0.74
Switzerland	1.21	1.12
China	0.24	0.67
Spain	0.63	1.03
United Kingdom	0.75	1.04
India	0.10	0.47
Italy	0.68	1.10
Japan	0.68	0.95
South Africa	0.23	0.63

Source: Penn World Table 9.0.

   a. Does the production function $y = k^{1/5}$ (shown in the table) generally underestimate or overestimate actual GDP per capita? What part of the production function do we usually assume explains these differences in predicted versus actual GDP?

This production function overstates actual GDP. We see that in most cases, the predicted GDP is greater than observed (actual) GDP.

Usually, we look to Total Factor Productivity (TFP), which is measured by A in the production function, to account for these differences. Countries use their resources in varying degrees of efficiency due to different technologies, institutions, etc.

b. What are two reasons why the production function in part 'a' tends to not accurately predict GDP per capita? (Hint: we went over three reasons in class).

In class we discussed the following possibilities:

1) Human capital differences – This refers to the stock of skills that individuals accumulate that makes them more productive. For example, different countries have different levels of education.
2) Institutions – Institutions are the rules, norms, and beliefs that shape behavior in a country. Property rights, legal system, social norms, etc.
3) Technology – Some countries are more technologically advanced than other countries, which enables them to produce more from the same amount of inputs.

c. Calculate the production function (Y) as a function of K and L?

$$y = k^{1/5} \rightarrow \frac{Y}{L} = \left(\frac{K}{L}\right)^{1/5} = \frac{K^{1/5}}{L^{1/5}}$$

If we multiply both sides of the equation by L, we obtain:

$$\frac{Y * L}{L} = \frac{K^{1/5}L}{L^{1/5}}$$

This simplifies to: $Y = K^{1/5}L^{4/5}$

3. State whether the following statement is true or false and provide a brief explanation and example:
"In a Cobb-Douglas production function, it is possible for a production function with constant returns to scale to have diminishing marginal product of labor and diminishing marginal product of capital."

The statement is true. Returns to scale has to do with the sum of the exponents on K and L, while marginal products have to do with the individual exponents on K and L. For constant returns to scale, the sum of the exponents has to be 1 and for diminishing marginal product each exponent has to be less than 1.
For example, the following production functions have both diminishing marginal products for K and L and constant returns to scale:

$$Y = \bar{A}K^{2/3}L^{1/3}$$
$$Y = \bar{A}K^{1/4}L^{3/4}$$
$$Y = \bar{A}K^{1/2}L^{1/2}$$

If a production function exhibits constant returns to scale, then the marginal products of capital and labor will be diminishing. However, diminishing marginal products does not guarantee constant returns to scale.

4. **Suppose you are given the following data for Cameroon:**
   Observed per capita GDP, relative to the United States, is 0.01
   Predicted per capita GDP, given by $y_P = k^{1/2}$, is 0.18.
   Provide an estimate of total factor productivity (A).

   If we are given that predicted GDP per capita is 0.18, then we can calculate what the level of capital per capita (k) is:

   $$y_P = k^{1/2} \rightarrow 0.18 = k^{1/2} \rightarrow k = 0.0324$$

   We plug this value into the production function where we set per capita GDP (y) equal to 0.01 and k equal to 0.0324:

   $$y = Ak^{1/2} \rightarrow 0.01 = A(0.0324)^{1/2} \rightarrow A = 0.056$$

   Note that your answer may be slightly different depending on how many decimals you rounded to.

5. **Suppose we are interested in estimating the effect of different institutions that a nation adopts on economic growth. Why can't we simply compare the different outcomes of any two countries (e.g. Venezuela and Canada) that have different institutions? Briefly explain.**

   If we want to compare the effect of a certain factor (e.g. institutions) on outcomes, we need to compare two groups (e.g. countries) that are nearly identical in every aspect other than adopting different institutions. If we just pick any two countries that have different institutions, they may be different in many other ways as well, and not just in the different institutions.

   That is why we went over the examples of West Germany versus East Germany and South Korea versus North Korea. Before their split, they were the same country and therefore it could be assumed that they are very similar. If they are very similar before adopting differing institutions, then we can argue that any difference (e.g. in GDP) after adopting differing institutions is due to differences in institutions.

6. **Suppose the production function of a country is $Y = 2K^{1/4}L^{1/4}$**

   a. **What does the 2 in the production function represent?**

   The 2 in the production function represents total factor productivity (A). This measures how productive countries are in using their inputs (K and L). The higher the value of A, the higher Y is going to be relative to any given values of K and L.

   b. **What is the marginal product of labor (MPL) equal to? Why is the optimal number of workers that should be employed obtained by setting MPL = w?**

   We can calculate the marginal product of labor by taking the partial derivative of the production function with respect to labor:

$$\frac{\partial Y}{\partial L} = 2K^{1/4}\,\frac{1}{4}L^{-3/4} = 0.5\,\frac{K^{1/4}}{L^{3/4}}$$

   The optimal number of workers is obtained by setting *MPL* equal to *w* because beyond that point, the *MPL* (how much workers produce) is going to be less than *w* (how much workers are getting paid). This means that hiring additional workers is decreasing the firm's profits because they are paying workers more than what they are producing. Up to this point where *MPL* = *w*, the amount paid to workers (w) is less than the value of what they are producing (MPL).