International Finance 4832
Lecture 3: Exchange Rates in the Long Run - PPP and Monetary Model

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The Exchange Rate in the Long Run: Purchasing Power Parity

This Lecture: Exchange Rates

This lecture - Exchange Rates Fundamentals (Chapter 13)

1. Last lecture:

UIP, CIP : (interest rate) Parity Conditions based on financial asset prices

- CIP: with Forward ER → explains the forward rate
- UIP: with Spot ER → explains the spot rate

2. Now: Chapter 14
   - Parity Conditions based on Goods Prices
   - LOOP: Law of One Price → for single goods
   - PPP: Purchasing Power Parity → for many goods

Short run: Rigid prices
Long run: Flexible prices

3. Later: Price levels and Monetary Approach to Exchange Rates in the long run (also Chp 14)

Law of One Price

LOOP: The price of an **identical good** in two countries should be the same when **expressed in a common currency**

Example: the dollar price of a coffee cup in US vs. the price, also in dollars, in the UK

If prices were not equal there could be arbitrage opportunities (in trading goods)

Let p^g_{us}: dollar price of good g in the US, p^g_{eu}: euro price of good g in France

Price in USD of good g (in Europe)

$$\text{LOOP:} \qquad p^g_{us} = p^g_{eu} \times E_{\$/€}$$

Price in USD of good g (in the US)

If it does not hold: say $p^g_{us} < p^g_{eu} \times E_{\$/€} \implies$ Buy in US and Sell in EU
(i.e. not an equilibrium yet, there is more trade and prices chances)

Should this hold for every good? $\rightarrow$ Not really, we need **several assumptions**:

Good should be Tradable, transportation costs low, market for the good is competitive (no monopoly power), and not too regulated (e.g., patents in pharmaceutical companies)

i.e., we need this good's market to be relatively "frictionless"

Law of One Price: Most Famous Application, the Big Mac Index

The Economist calculates the Big Mac Index: compare the LOOP implied price of a big mac with the actual price to gauge whether the actual ER is over/under valued

Why Big Mac? → because the assumptions hold relatively well (very simple good)

Now we can compare the ER implied by the LOOP vs the actual one

LOOP Implied FX rate: $E_{F/\$}^{(\text{implied})} = P_{foreign}/P_{us} \Rightarrow$ (Over/Under) Valuation $=$ (Implied ER/Actual ER) $- 1$

If the valuation is negative then the ER is undervalued (actual ER should be lower)

Same as $\frac{1}{E_{\$/F}^{(\text{implied})}}$

	local currency price	fx rate (fx/$)	dollar price	implied fx rate	over/under valuation
United States	4.9	1.00	4.93		
Argentina	33.0	13.81	2.39	6.69	–52.0
China	17.6	6.56	2.68	3.57	–46.0
Norway	46.8	8.97	5.21	9.49	??

$\frac{33}{13.81} = 2.39$

$\frac{33}{4.93} = 6.69$

$\frac{6.69}{13.81} - 1$

e.g., for China, $E_{ch/\$} = 17.6/4.9 = 3.57 \implies$ Yuan is undervalued (actual is 6.56 yuan per dollar)

↪ Implied by LOOP

Purchasing Power Parity

Generalization of LOOP to a basket (bundle) of Goods

More interesting as we measure inflation in terms of a consumption basket

The parity refers to the **same basket** in both locations

Let: P_{us}: dollar price of basket of goods in US, P_{eu}: euro price of basket of goods in EU

(Absolute) ↳ Levels

Cost of life in USD (in Europe)

PPP: $P_{us} = P_{eu} \times E_{\$/€}$

Cost of life in USD (in the US)

This is the Absolute PPP: Absolute because is expressed in terms of Prices Levels
(relative will refer to inflation or growth rates)

If the LOOP holds for each good in the basket the PPP holds

(if LOOP does not hold for some goods the PPP may still hold though, PPP averages prices)

Relative PPP

Relative Purchasing Power Parity: PPP in growth rates

Prices growth rates: inflation; ER growth rate: depreciation

Now we must keep track of the timing (to measure the growth)

For $t+1$: $P_{us,t+1} = P_{eu,t+1} \times E_{\$/€,t+1}$

For t: $P_{us,t} = P_{eu,t} \times E_{\$/€,t}$

Then (one expression divided by the other):

$$\frac{P_{us,t+1}}{P_{us,t}} = \frac{P_{eu,t+1}}{P_{eu,t}} \times \frac{E_{\$/€,t+1}}{E_{\$/€,t}}$$

Now, apply logs, or notice that $\frac{P_{us,t+1}}{P_{us,t}} = 1 + \pi_{us}$, also, let the depreciation rate be $d_{\$/€}$.

The expression above is then: $(1 + \pi_{us}) = (1 + \pi_{eu})(1 + d_{\$/€})$

then: $\pi_{us} = \pi_{eu} + d_{\$/€}$

That is, the depreciation is given by the differential in inflation rates:

$$\pi_{us} - \pi_{eu} = d_{\$/€}$$

Relative PPP (cont)

Relative PPP: If inflation is higher at home the home currency **depreciates**

This is intuitive: the home currency is less valuable now (can buy fewer goods)

$$\text{[Relative PPP]}: \qquad \pi_{us} - \pi_{eu} = d_{\$/€}$$

in other words: ER grows according to the Inflation differentials across locations

Example: $\pi_{us,t} = 4\%$ and $\pi_{mx,t} = 1.5\%$

Then: $4\% - 1.5\% = 2.5\%$ ... the Dollar depreciates and the Peso appreciates

Example 2: $\pi_{us,t} = 4\%$ and $\pi_{tk,t} = 10\%$

What happens to the US dollar and to the lira?

$\pi_{us,t} - \pi_{tk,t} = d_{\$/tk,t}$

$4\% - 10\% = -6\%$

} USD appreciates, Lira depreciates

Relative PPP: Predictions

Prediction made by the Relative PPP:

- For countries with higher inflation $\rightarrow$ the (home) currency depreciates
- For countries with lower inflation $\rightarrow$ the currency appreciates

$$\pi_{us} - \pi_{eu} = d_{\$/€}$$

PPP: Evidence

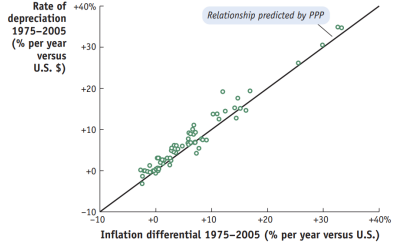
How to tell whether the Relative PPP holds in the data?

Plot each side of the main equation:

$$\pi_{us} - \pi_{eu} = d_{\$/€}$$

If the Relative PPP holds, the data points should lie on a 45-degree line
(as each side of the equation is supposed to be equal to the other)

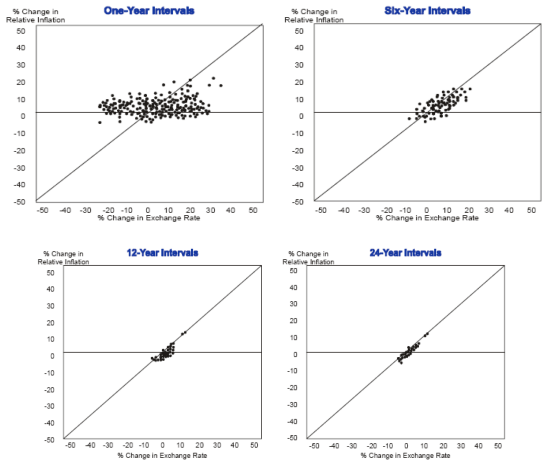
Figure: Inflation differentials and ER 1975-2005



Does it always work?

No, the **PPP holds relatively well in the Long Run** (when prices are able to adjust) but is not a good approximation for the Short Run

Figure: Inflation differentials at different horizons



Summary

Law of One Price:

- Individual goods should have the same price in different locations (in same currency)
- Holds for some goods, not for others
- depends on how tradable the good is (and other market assumptions)

Purchasing Power Parity (PPP):

- Works well in the Long Run (long horizons)
- Adjustment of Prices is slow (sticky prices)
- Half life of prices gap: Around 4 years $\implies$ does not work for Short Horizons

In the long run: Inflation Differentials determine the Exchange Rates

We'll see next what determine the Prices (inflation) differentials
(for that we must think about money and monetary policy)

The Monetary Approach to the Exchange Rate

Part 1

Road ahead

Before: the PPP $\rightarrow$ ER can be tied to Prices differentials (or inflation in Relative PPP)

But how are Prices (inflation) determined?

- We will analyze price determination. For that we consider the **quantity theory of money**
- Modelling Pieces: PPP + Quantity Theory of Money
- Include interest rate as an extra piece (making money demand sensitive to int. rate changes)
- Results: Real Interest Rate Parity (so far parities —CIP/UIP— were set in terms of nominal rates)

PPP: Ties ER to Prices Quantity Theory: Prices as function of Money and Real income

Later: Exchange Rates in the Short Run (chapter 15)

Purchasing Power Parity

Absolute Purchasing Power Parity (PPP) condition:

Let P_{us} : dollar price of *basket* of goods in the US; P_{eu} : euro price of *basket* of goods in the EU

$$\textbf{[PPP]:} \qquad P_{us} = P_{eu} \times E_{\$/€}$$

Relative Purchasing Power Parity: PPP in growth rates

$\pi_{us,t}$: inflation rate in the US in period t; $\pi_{eu,t}$: inflation rate in Europe (EU) in period t

$$\textbf{[RPPP]:} \qquad \pi_{us,t} - \pi_{eu,t} = d_{\$/€,t}$$

Money and Prices:

Given these parities, we have to explain what drives the prices and inflation

We use the Quantity Theory of Money for this (how's money defined?)

Money

Money: instrument or device with the following properties:

1. Unit of account
2. Store of value
3. Medium of exchange (used for transactions)

Trade-off: 2 vs. 3: Not a great store of value (yields zero interest rate), best instrument for transactions

Central Bank Supplies money: we assume it controls the Money Supply accurately.

Quantity Theory of Money

Assumption: demand for money is proportional to nominal income (prices times real income)

Why? $\rightarrow$ with higher prices you need more money to pay for the same goods, or with higher real income you would like to buy more goods/services.

$$M^d = \bar{L}PY$$

Here, M^d: money demand, $\bar{L}$: liquidity demand (constant for now), P: price level, Y: real income

What happens with the quantity of money demanded if the real income increases by 10%?

Equilibrium level of Money

The Central Bank determines the Money Supply: M^s

Nominal Money Demand: $M^d = \bar{L}PY$

In equilibrium (supply = demand) we have: $M^s = M^d = M$

(that's why most of these models just use M and refer to them as "money supply" or "demand" as if they were the same ... in equibrlium, they are)

$$M = \bar{L}PY$$

Rearrange to find the price level:

$$P = \frac{M}{\bar{L}Y}$$

Monetary Theory of Prices:

Price determined by how much nominal money is issued relative to the Real Income (or real GDP)

Quantity Theory + PPP: Explaining the Exchange Rate

Now we can use one price equation for each country and see what the ER depends on:

$$P_{uk} = \frac{M_{uk}}{\bar{L}_{uk} Y_{uk}} \qquad P_{us} = \frac{M_{us}}{\bar{L}_{us} Y_{us}}$$

Substitute in the PPP:

$$E_{\$/£} = \frac{\frac{M_{us}}{\bar{L}_{us} Y_{us}}}{\frac{M_{uk}}{\bar{L}_{uk} Y_{uk}}} = \frac{M_{us}/M_{uk}}{\bar{L}_{us} Y_{us}/\bar{L}_{uk} Y_{uk}}$$

All else equal, suppose M_{us} doubles. What happens to P_{us} and to $E_{\$/£}$ why?

Quantity theory + Relative PPP

Result from before was obtained with Absolute PPP (in levels)

Now we extend it to growth rates: Relative PPP and Money Supply growth

First, express quantity theory equation in growth rates:
(remember, many ways, e.g., using logs for equations in different times, or expressing rate of changes of numerator minus those of the denominator)

$$P_{us,t} = \frac{M_{us,t}}{\bar{L}_{us} Y_{us,t}} \qquad \longrightarrow \qquad \pi_{us,t} = \mu_{us,t} - g_{us,t}$$

π: inflation rate, μ: money growth rate, g: real income growth rate.

Quantity theory + Relative PPP (cont.)

Relative PPP:

$$\pi_{us,t} - \pi_{uk,t} = d_{\$/£,t}$$

Use a quantity theory equation for each location:

$$\pi_{us,t} = \mu_{us,t} - g_{us,t} \qquad \pi_{uk,t} = \mu_{uk,t} - g_{uk,t}$$

Subtract one equation from the other:

$$\pi_{us,t} - \pi_{uk,t} = (\mu_{us,t} - g_{us,t}) - (\mu_{uk,t} - g_{uk,t}) = d_{\$/£,t}$$

Rearrange by similar terms:

$$\underbrace{(\mu_{us,t} - \mu_{uk,t})}_{\text{Diff in Money Growth}} - \underbrace{(g_{us,t} - g_{uk,t})}_{\text{Diff in real growth rates}} = \underbrace{d_{\$/£,t}}_{\text{ER depreciation}}$$

Quantity theory + Relative PPP (cont.)

ER dynamics (depreciation) given by:

$$\underbrace{(\mu_{us,t} - \mu_{uk,t})}_{\text{Diff in Money Growth}} - \underbrace{(g_{us,t} - g_{uk,t})}_{\text{Diff in real growth rates}} = \underbrace{d_{\$/£,t}}_{\text{ER depreciation}}$$

Thus:

Higher cross-country money growth differentials increase the depreciation rates

Higher cross-country real income growth differentials decrease the depreciation

Example: US money growth: 2% per year, UK money growth: 5% per year. US real economy (income) growth: 3% per year, UK real economy growth: 1.5% per year.

What happens to the dollar? pound?

$$(2\% - 5\%) - (3\% - 1.5\%) = -4.5\%$$

USD appreciates, GBP depreciates

Forecasting foreign exchange rates

UIP arbitrage requires a forecast or guess of the future ER: $E^e_{\$/£}$

If we have forecasts for money growth and GDP (income) we can use:

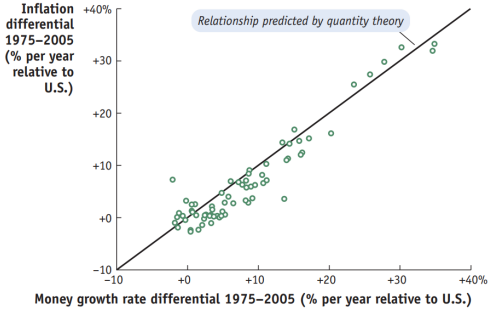
$$(\mu_{us,t+1} - \mu_{uk,t+1}) - (g_{us,t+1} - g_{uk,t+1}) = d_{\$/£,t+1}$$

These forecasts can become available. The first depends on what the Central Bank does (and announces), the second on how the economy is performing.

Critical for using this approach: How well does the PPP and Quantity Theory hold?

These theories in practice

Figure: Evidence for Monetary Approach: Quantity Theory and Inflation



It seems to work . . . but notice the time frame (30 years)

For shorter horizons (short run) PPP does not work well
(why? ... because prices do not adjust fast enough (sticky prices))

PPP: Critical building block here. Thus this approximation is better for Long Run analysis

PPP, the Quantity Theory and Hyperinflations

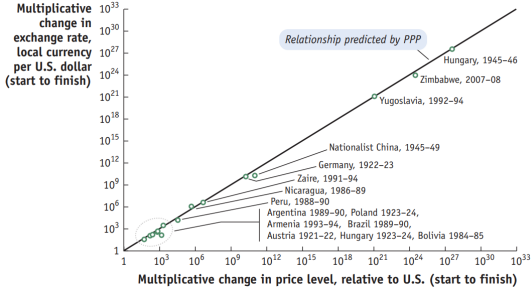
PPP + Quantity theory works well when prices do adjust $\rightarrow$ that is, in the Long Run

One Exception: Hyperinflations

- Episodes of high inflation increases: about 20% per month or more (can be way more)
- For inflation to move like that within a month prices must become quite flexible shortly

This is how we know (empirically) that this theory's weakness in the short run relates to price rigidities

Figure: Evidence for Monetary Approach: Hyperinflations



PPP, the Quantity Theory and Hyperinflations (cont.)

Hyperinflations help us see how quantity theory works: prices are flexible and move according to their expected relationship with other variables

Hyperinflations show us one more thing: L can change!

Remember: L or $\bar{L} \rightarrow$ responsiveness of real money to real income ($M/P = LY$)

also, L: Liquidity Demand

We assumed L is constant: It is not

- Opportunity cost of holding money is high and can change (bad store of value)
- Increases with interest rate

Thus, the higher the interest rate, the lower L as you want to hold less cash

Quantity theory with Interest-sensitive Liquidity

Before:

$$M^d = \bar{L}PY$$

Now (more complete model):

$$M^d = L(i)PY$$

$L(i)$ decreases with the nominal interest rate i

- Money has a nominal interest rate of zero while other assets (e.g. bonds have a higher return
- Opportunity cost of money: i (interest rate)

Now we extend the for the ER (monetary approach) with $L(i)$ instead of $\bar{L}$

Summary

- PPP ties prices to exchange rates (levels and changes)
- Quantity theory ties money to the price level and money growth to the inflation
- Together they explain the ER depreciation:

$$(\mu_{us,t} - \mu_{uk,t}) - (g_{us,t} - g_{uk,t}) = d_{\$/£,t}$$

  Higher money growth in the US $\rightarrow$ dollar depreciates

  Higher real growth in the US $\rightarrow$ dollar appreciates

- Model works well when prices are flexible $\rightarrow$ in the Long Run (or in hyperinflations)
- Better version of the model requires allowing liquidity demand to change (with interest rate)

The Monetary Approach to the Exchange Rate

Part 2: Model with variable Liquidity

Roadmap

Before:

- CIP, UIP: Interest Rates Parities (Asset prices based conditions)
- PPP: Goods prices based parity condition
- Monetary approach to the ER: Quantity theory of money + PPP

Now: Exchange Rates in the Long Run (PPP based theory)

- Add interest-sensitive money demand ($L(i)$) to quantity theory of money
- Real interest parity
- Central Bank policies and targets

Later: Exchange rates int he short run (chapter 15)

Quantity theory with Interest-sensitive Liquidity

Before:

$$M^d = \bar{L}PY$$

Now (more complete model):

$$M^d = L(i)PY$$

$L(i)$ decreases with i

i (nominal interest rate): opportunity cost of money

Nominal and Real Interest Rates

Nominal interest rate (i) is the return to saving (or cost of borrowing) **in terms of money**
(e.g., if i=6% per year, a one-year loan returns 6% more dollars)

Real interest rate (r) is the return to saving (or cost of borrowing) **in terms of purchasing power**
(e.g., if i=4% per year, inflation is 2%, then a one-year loan returns 2% more consumption capacity)

Difference between the two: (expected) inflation $\rightarrow \pi^e$

$$i_t = \pi_t^e + r_t$$

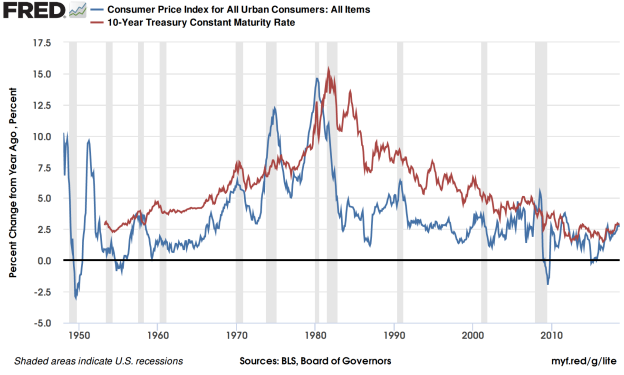
The Fisher Effect

$$i_t = \pi_t^e + r_t$$

This is the *Fisher equation* and allows us to formulate a known result, the *Fisher effect*

Fisher effect: All else equal a rise in (expected) inflation is met with an equal rise in nominal rate

Figure: the Fisher Effect: Inflation vs. Nominal interest rate in the US



Nominal and real interest rates (cont.)

Suppose $i_t = 6\%$, $\pi_t^e = 2\%$, then $r_t = 4\%$

$$i_t = \pi_t^e + r_t \tag{1}$$
$$6\% = 2\% + 4\% \tag{2}$$

Now, notice π_t^e is an expectation. Actual inflation could be different.

(unexpected) Inflation: good for borrowers, bad for lenders

Inflation uncertainty makes borrowing/lending risky and less frequent

Thus, keeping inflation predictable and stable is an important goal of central banks.

Using the nominal interest rate

We can combine the PPP and UIP

The latter brings the nominal rate into our modeling mix

$$[UIP:] \qquad d^e_{\$/€} = i_{\$} - i_{€}$$

We can write the PPP in expectations (take expectations at each side of the equation):
(the condition should hold in the present, or for the unknown future and based on expectations)

$$[PPP:] \qquad d^e_{\$/€} = \pi^e_{us} - \pi^e_{eu}$$

Remember: PPP $\rightarrow$ no arbitrage in goods market, UIP $\rightarrow$ no arbitrage in asset markets

In the Long Run both tend to be correct. Now let's equalize these eqs:

$$i_{\$} - i_{€} = \pi^e_{us} - \pi^e_{eu}$$

Real Interest Parity

Rearrange this equation (terms of each country on each side):

$$i_{\$} - i_{€} = \pi^{e}_{us} - \pi^{e}_{eu} \tag{3}$$

$$i_{\$} - \pi^{e}_{us} = i_{€} - \pi^{e}_{eu} \tag{4}$$

$$r_{us} = r_{eu} \tag{5}$$

We have obtained another parity: Real Interest rates should be equal across locations

Note: trading of assets and goods $\longrightarrow$ convergence in real rates (assumes flexible prices)

This should hold for any country pair. Thus real interest of country i is: $r_i = r^*$

Where r^* : world interest rate (exogenous to any country)

Exogenous means that can be treated as given (fixed or constant) by any particular country
(nothing they do will change r^*)

Fisher Effect - revisited

If r is the same across countries (equal to r^*) then:

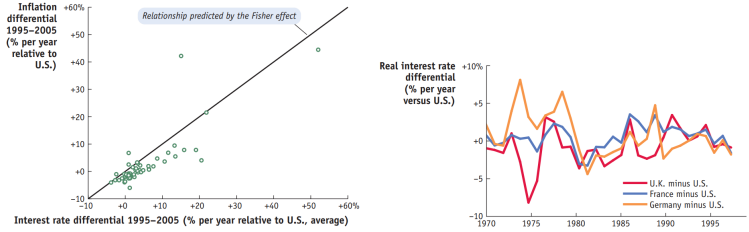
$$i_{\$} = \pi^e_{us} + r^* \qquad i_{€} = \pi^e_{eu} + r^*$$

Differences in nominal interest rates reflect differences in inflation

(take one eq. subtract the other, the r^* cancels out)

Does this hold?

Figure: the Fisher Effect: Inflation vs. Nominal interest rate in the US



Holds only in the Long Run: real rate differences are not zero but converge to zero over time.

Monetary Model with liquidity demand

$$M^d = L(i)PY$$

$L(i)$: decreasing function of i (i: opportunity cost of money) Demand for real balances:

$$\frac{M^d}{P} = L(i)Y$$

Demand for real balances now moves when:

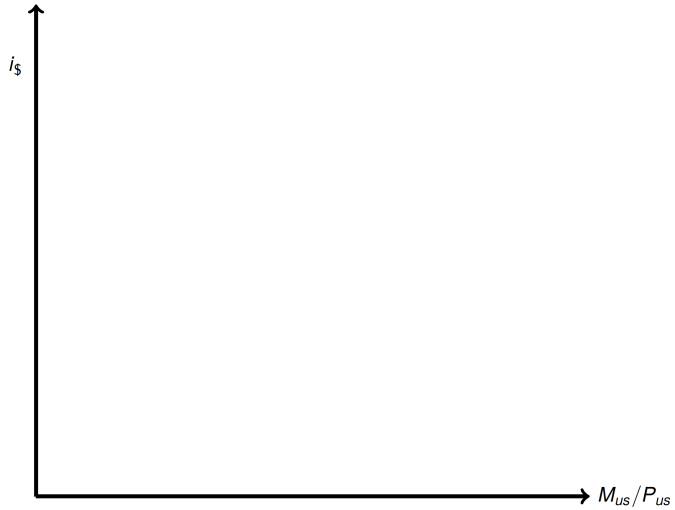
i changes (new)

Y changes

(or either/both)

Different approaches to model money

Figure: Real money and nominal interest rate on each model



Monetary Approach to ER (with interest-sensitive liquidity)

Use PPP + Quantity theory but now with $L(i)$:

$$E_{\$/£} = \frac{P_{us}}{P_{uk}} = \frac{\frac{M_{us}}{L(i_{us})Y_{us}}}{\frac{M_{uk}}{L(i_{uk})Y_{uk}}} = \frac{M_{us}/M_{uk}}{L(i_{us})Y_{us}/L(i_{uk})Y_{uk}}$$

Let λ be the growth rate of $L(i)$, then in growth rates we have:

$$(\mu_{us,t} - \mu_{uk,t}) - (g_{us,t} - g_{uk,t}) - (\lambda_{us,t} - \lambda_{uk,t}) = d_{\$/£,t}$$

Before $\lambda = 0$; Now: $\lambda = 0$ if i is constant (does not change), $\lambda < 0$ if i increases

Effects of money growth

Let $g_{us} = g_{uk} = 0$ and all variables in the UK unchanged (M_{uk}, i_{uk},etc)

Money growth in the US is constant until a date T when it increases and becomes $\mu_{us} + \Delta\mu_{us}$

Let's see what happens in M, M/P, $i_{\$}$, P_{us}, $E_{\$/£}$ through the lenses of each model

Model with constant liquidity demand ($\bar{L}$)

Model with variable liquidity demand ($L(i)$)

Taking stock: the role of expectations

The model with $L(i)$ generates **more volatile** inflation and exchange rates (they jump as new information comes in!)

This is the result of a **change in expectations**

1. At T agents learn that future inflation will be higher
2. They foresee depreciation of the dollar in the future (by PPP)
3. And sell dollars for euros (starting now)
4. The Dollar instantly depreciates, even though nothing has changed yet

That's why variables jump discontinuously at T

Remember: Spot ER (and other financial asset prices) are quite volatile $\rightarrow$ they react constantly to (changes in) expectations.

Central Bank policy targets

Policy objective: predictable and stable inflation (prices)

Uncertainty $\implies$ changes in expectations $\implies$ too volatile inflation

How does a central bank manage people's expectations?

that is, how do they convince the public that inflation will be stable (policy jargon: "anchor expectations")

One way: have a great reputation (e.g., Fed, ECB)

Another way: Use nominal anchors (i.e., abide by a constraint or a rule)

1. Exchange rate target
2. Money supply target
3. Inflation target

Intuition: bank commits to set a variable in a given way (target its value) that keeps prices stable

Exchange rate target

From Relative PPP $\longrightarrow$ anchor: ER

$$\pi_h = d_{h/f} + \pi_f$$

Target a level for $d_{h/f}$. Set monetary policy to make sure the target is met.

Idea here: stabilize (or fix) the ER with respect to a stable inflation economy

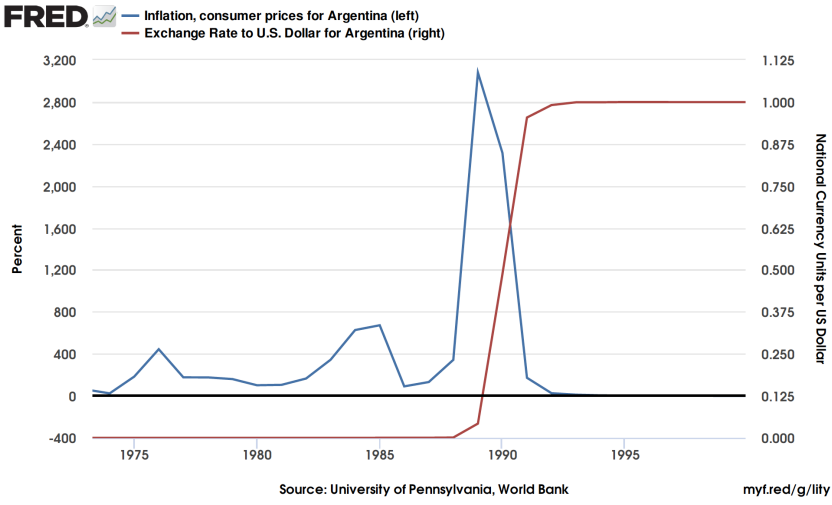
Most extreme case: $d_{h/f} = 0$

Then: $\pi_h = \pi_f$ ... that is: home country "imports" inflation from another economy

This is what Argentina did to stop a hyperinflation episode

Exchange rate target (cont.)

Figure: Inflation and Exchange Rate: Argentina



Money supply target

Quantity theory $\longrightarrow$ anchor: Money Supply

$$\pi_h = \mu_h - g_h - \lambda_h$$

Set μ_h at a constant or stable level (target). Use policy make sure the target is met.

Disadvantage: g_h and λ_h still move around, so π_h may not be stable yet, even after setting the target!

Due to this flaw the scheme is not applied widely (the least popular of the three)

Inflation target

Fisher equation $\longrightarrow$ anchor: i_h (nominal interest rate)

Target an inflation level π_h. Adjust i_h to meet the target.

r^* is roughly constant (global interest rate)

Advantage: Central banks have good control over $i_h \implies$ control over π_h

Many central banks to this: between the 90's and up to 2010 the inflation targeting countries went from 8 to 54 (source: `centralbanknews.info`)

Summary

- PPP + UIP $\Longrightarrow$ Real interest rate parity (real rates converge)

  - Inflation differences lead to nominal interest rate differences

- Include interest-sensitive liquidity demand in quantity theory
- predictions are similar to simpler model. But **changes in expectations** induce more volatility (in prices, ER, etc)
- Expectations are key for policy makers' purposes $\longrightarrow$ Nominal anchors and targets as expectations' control tools
  - Targets: (from PPP) ER, (from Quant. theory) Money Supply, (from Fisher) Inflation