Problem Set # 4

Answer Key

Answer the following questions. Show your work. As mentioned in class, you are encouraged to work in groups but must write your own answers.

1. **(Based on FT 17.1 - External Wealth Changes)** Answer the following questions on the changes in External Wealth (use the equations and notations discussed in class and the chapter 17 of FT). Assume the NFIA is zero, no capital gains on external wealth, and no unilateral transfers.

   (a) Express the change in external wealth (ΔW_0) at the end of period 0 as a function of the economy's trade balance (TB), the real interest rate (a constant r^*), and initial external wealth (W_{-1}).

   Given the assumptipns $NFIA_0 = 0$, $NUT_0 = 0$, $KA_0 = 0$ and then $CA_0 = TB_0$, also capital gains $= 0$

   With that the change in wealth is the trade balance (new assets) plus the income proceeds (returns) on the previous wealth stock in $t = -1$

   $$\Delta W_0 = TB_0 + r^* W_{-1}$$

   (b) Using the result in (a), write an expression for the stock of external wealth at the end of period 0 (W_0). This should be written as a function of the economy's trade balance (TB_0), the real interest rate (r^*), and initial external wealth (W_{-1}).

   From (a) we have that $W_{-0} - W_{-1} = \Delta W_0 = TB_0 + r^* W_{-1}$, then we just solve for W_0:

   $$W_0 = TB_0 + (1 + r^*) W_{-1}$$

   (c) Using (a) and (b), write an expression for the stock of external wealth at the end of period 1 (W_1). This should be written as a function of the economy's trade balance (TB) each period, the real interest rate, and initial external wealth.

   The wealth at the end of year 1 has the same form as depicted previously:

   $$W_1 = TB_1 + (1 + r^*) W_0$$

Now we need to substitute the result we obtained in (b): $W_0 = TB_0 + (1 + r^*)W_{-1}$:

$$\begin{aligned} W_1 &= TB_1 + (1 + r^*)W_0 \\ &= TB_1 + (1 + r^*)\left(TB_0 + (1 + r^*)W_{-1}\right) \\ &= (1 + r^*)TB_0 + TB_1 + (1 + r^*)^2 W_{-1} \end{aligned}$$

(d) Using your answers from (a), (b), and (c), write an expression for the stock of external wealth at the end of period 2 (W_2). This should be written as a function of the economy's trade balance (TB) each period, the real interest rate, and initial external wealth.

The wealth in period 2 follows the same formula, we just need to substitute in it the W_1 we obtained in (c) (which implicitly implies substituting all previous levels of wealth we know).

$$\begin{aligned} W_2 &= TB_2 + (1 + r^*)W_1 \\ &= TB_2 + (1 + r^*)\left((1 + r^*)TB_0 + TB_1 + (1 + r^*)^2 W_{-1}\right) \\ &= (1 + r^*)^2 TB_0 + (1 + r^*)TB_1 + TB_2 + (1 + r^*)^3 W_{-1} \end{aligned}$$

(e) Suppose we require that W_2 equals zero. Write down the condition that the three trade balances (in periods 0, 1, and 2) must satisfy. Arrange the terms in present value form.

The terminal wealth is zero $W_2 = 0$, we can replace that in the result we obtained in (d) and rearrange the terms to equal the present value of the trade balances and minus the present value of the initial wealth. These "present" values are expressed with respect to period 0 (but could be stated with respect to any of the other periods):

$$\begin{aligned} W_2 &= (1 + r^*)^2 TB_0 + (1 + r^*)TB_1 + TB_2 + (1 + r^*)^3 W_{-1} \\ 0 &= (1 + r^*)^2 TB_0 + (1 + r^*)TB_1 + TB_2 + (1 + r^*)^3 W_{-1} \\ -(1 + r^*)^3 W_{-1} &= (1 + r^*)^2 TB_0 + (1 + r^*)TB_1 + TB_2 \\ -(1 + r^*)W_{-1} &= TB_0 + \frac{TB_1}{(1 + r^*)} + \frac{TB_2}{(1 + r^*)^2} \end{aligned}$$

The last line is obtained by dividing each side by $(1 + r^*)$. The last expression has the negative of the present value of wealth on the left-hand side of the equation and the present value of the trade balance flows on the right-hand side.

2. **(External Wealth and Current Account)** Consider a country with no initial wealth that exists for two periods. The country can produce 100 units of output in the first period and 120 units of output in the second period. The country can borrow or lend on world markets at a world real interest rate of 5 percent. The household has the utility function $u = \min(C_0; C_1)$ and C_t is the household

consumption in period t.

Assume there is no government expenditure, nor investment ($G = I = 0$).

(a) Solve for the level of gross national expenditure in both periods. Assume there is no government spending nor investment. [Hint: replace $TB = GDP - GNE$ in the LRBC and check closely the relation between C and National Expenditure under our assumptions, also remember that consumption smoothing is optimal in this case]

With no initial wealth the budget constraint is

$$0 = TB_0 + \frac{TB_1}{1 + r^*}$$

We can replace $TB = GDP - GNE$

$$0 = GDP_0 - GNE_0 + \frac{GDP_1 - GNE_1}{1 + r^*}$$

Additionally, we know that $GNE = C$, we can also substitute the interest rate $r^* = 0.05$ and the output value (100)

$$0 = 100 - C_0 + \frac{120 - C_1}{1.05}$$

Rearranging, the present value of consumption flows is equal to that of the output flows:

$$C_0 + \frac{C_1}{1.05} = 100 + \frac{120}{1.05}$$

Finally, we use the solution to the utility maximation: $C_0 = C_1 = C$ and replace $C_0,\ C_1$:

$$C + \frac{C}{1.05} = 100 + \frac{120}{1.05}$$

Solving for C: $C = 109.756$. That means $GNE = 109.756$ in each period.

(b) What is the current account balance (and its components), and what is the financial account balance in each of the two periods? Assume no unilateral transfers and a capital account of 0 in both periods when answering ($KA = 0,\ NUT = 0$).

$$TB_0 = Q_0 - C_0 = 100 - 109.756 = -9.756$$
$$CA_0 = TB_0 + NFIA_0 = -9.756 - 0$$
$$TB_1 = Q_1 - C_1 = 120 - 109.756 = 10.244$$
$$NFIA_1 = 0.05(-9.756) = -0.488$$
$$CA_1 = TB_1 + NFIA_1 = 10.244 - 0.488 = 9.756$$
$$FA_0 = 9.756$$
$$FA_1 = -9.756$$

3. **(Cross border investment and productivities)** Germany has the production function $q_G = 30k_G^{1/3}$, where q is output per worker and k is capital per worker. Brazil has the production function $q_B = 15k_B^{1/3}$.

   (a) If $k_G = 1000$ and $k_B = 900$, which country has a higher output per capita?

$$q_G = 30k_G^{1/3} = 30(1000)^{1/3} = 300$$
$$q_B = 15k_B^{1/3} = 15(900)^{1/3} = 144.8$$

The output per-capita is higher in Germany

   (b) Would you expect to see Germany investing in Brazil or Brazil investing in Germany? Explain your answer.

We can get the marginal product of capital in each case and compare ($MPK = \frac{\partial q}{\partial k} = A\frac{1}{3}q^{-2/3}$ where A is the productivity or TFP):

$$MPK_G = 10(1000)^{-2/3} = 0.1$$
$$MPK_B = 5(900)^{-2/3} = 0.054$$

The productivity of capital is almost double in Germany, thus you would expect capital to flow out of Brazil and towards Germany (Brazilians investing in German projects)

   (c) Suppose Germany country imposed a tax on foreign interest payments of 5 percent. Would this change your answer to part (b)? Explain your answer.

The return after taxes is: $10\% - 5\% = 5\%$, which is lower than what Brazilians can get by investing at home (5.4%).

Given that, it is not appealing for Brazilians to invest in Germany anymore and we would expect no cross-border investment flows in that direction.

4. **(IS-LM-FX)** Suppose the (Home) economy is initially in equilibrium and there is a sudden increase in the interest rate of the foreign country i^*. Use the IS-LM-FX model to answer the following questions. Assume the home country has a *floating exchange rate regime*. Provide plots with your answers.

   (a) How the change in i^* affects the new equilibrium values of the home interest rate (i), output (Y), and spot exchange rate $E_{h/f}$? Does the home currency appreciate or depreciate?

The increase in i^* shifts up the FR curve (foreign return) in the FX market as ROW returns are higher for any spot rate. As a result the exchange rate depreciates (increases). This raises the trade balance shifting the IS curve upwards. This increases the equilibrium home rate which increases the DR curve in the FX market (domestic return).

The final equilibrium is the intersection of a higher IS curve, the same LM curve as before, and higher DR and FR curves. The final equilibrium values and change with respect to the initial equilibrium are: Y_2 (increased due to higher trade balance), i_2 (increased), E_2 (increased). Then, the ER depreciates (see plots at the end).

(b) Explain the effect in the home trade balance.

The trade balance increased due to the depreciation of the exchange rate. See plots at the end.

(c) Now assume that the home country has a *fixed exchange rate regime*. How does the central bank responds to the change in i^*

The central bank must prevent the exchange rate from changing. It will intervene by pushing the interest rate up to match the change in the foreign rate. This is done with a money supply contraction (shifts the LM curve to the left). See plots at the end.

(d) How does the home interest rate (i), output (Y), and spot exchange rate $E_{h/f}$ change in this case? Show the new equilibrium values with plots.

The FR curve shifts up due to the higher foreign rate. The home central bank, fearing the depreciation pressure, will contract the money supply, shifting the LM curve to the left and increasing the home rate. This shifts the DR curve up which undoes the changes in the exchange rate.

At the end, the final exchange rate is the same as before (fixed rate), output decreases and the home rate increased. See plots at the end.

(e) When is an economy that follows a fixed exchange rate regime less likely to commit to this policy (and increase the interest rate at home). Pick one and explain your answer.

i. During a recession at home an a boom in the foreign country

ii. During a recession at home an a recession in the foreign country

iii. During a boom at home and a boom in the foreign country

iv. During a boom at home and a recession in the foreign country

Answer: During a recession at home an a boom in the foreign country.

The home country is more likely to abandon the fixed rate regime when it makes it raise interest rates during a recession. This is more probable during a boom in the foreign country. In that case the foreign country wants to set high interest rates (to cool down their economy) and the home economy has a much harder time mimicking their policies since they have to manage their recession.

Figure 1: Plots for 4A,B

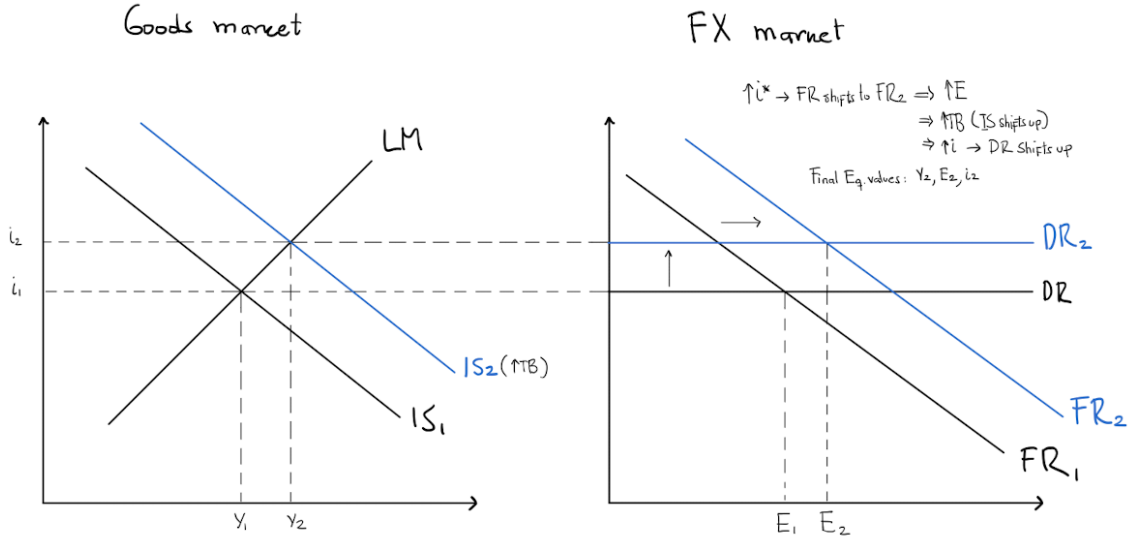


Figure 2: Plots for 4C

