Paper Report: Financial Cycles and Heterogenous Intermediaries

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Summary

The authors provide a general equilibrium framework that allows to model financial heterogeneity across banks in leveraging. Such heterogeneity comes in the form of different Value at Risk constraints that limit the maximum borrowing the banks can absorb.

The inclusion of the heterogeity let the authors study the economic effects of the cross section of bank types, classified by the risk profile of their balance sheets.

The model has a number of simplifications to preserve tractability. However, monetary policy, in the form of a subsidized interest rate, is included. This addition is important to analize the general equilibrium effect of monetary policy expansions.

Theoretical Benchmark (Prior) : The frictionless case would imply that, just as with capital acumulation and output, the effect of monetary policy on leverage and risk is mononotic and has no ulterior relation to the cross section of banks.

The benchmark framework would be a representative agent, representative bank with unlimited liability and therefore, with no need for government transfers or even a central bank. In a frictionless world, this would correspond to a standard household, firm, financial intermediary (competitive) jointly utility maximization subject to economic feasibility constraints.

Paper proposal: The authors include banks heterogeneity with limited liability and check what is the effect, on macroeconomic and risk dynamics, of entry/exit decisions in the risky capital market as well that of the amount of leverage banks intake when using external funding.

The framework makes necessary (given the limited liability) to include a government that guarantees the payments of returns to the depositors. Additionally, the authors add a central bank rule and a related shock, allowing to model the dynamic effect of monetary policy expansions in the economy and the leverage decisions across financial intermediaries types.

Result: The Extensive margin of banks (how many participate) and the Intensive margin (by how much they lever their activies) interplay so as to generate a non-linear effect of monetary policy in leverage and risk.

The outcome is a scale dependent trade-off between monetary policy and financial stability: When the Interest Rates are low, expanding the economy induces a higher financial crisis risk. With higher interests rates, doing so will bear no conflict and instead will lower the implied risks.

Extensions

The authors provide a number of directions to take. The model itself is very simplistic, but some of these simplications are not realistic. The number of suggested extensions comes either in the direction of improving an assumption or obey to the need of further scrutiny and explorations with the model.

We will focus on how to complement the model in this section and leave the suggestions on the modification of some assumptions for the referee report.

Future research and explorations regarding this model. May make necessary to consider:

- *Nominal version of the model*: The monetary policy goals are not restricted to expanding the output of the economy in times of crisis and dealing with productivity shocks. Inflation considerations and the exploration of the found trade-off in light of specific goals of the central bank can be important. Additionally, the interaction effects that can arise from considering nominal rigidities as those coming from sticky prices and wages are not neglictible and may induce to a less sharp decline in the returns of capital than shown.

- *Open economy model*: the inclusion of international monetary spillovers or global banks that circunvent the local regulations, implied by the VaR restrictions, may change completely the results of the model. New sources of funding, exchange rate risks, terms of trade effects and international capital flows may change the nature and direction of the monetary policy - financial stability framework. It's hard to tell concretely how this occurs and therefore it may be worth exploring.

- *Richer structure of assets*: New assets like bonds may help create additional sources of funding as well as to allow hedging new sources of risk arising in more complete versions of the model. For example, in a model that considers the exchange rate, its possible to use bonds to hedge the associated risks.

- *Welfare evaluation of the model*: A normative evaluation of the model is needed to establish the optimal decision a central bank should take when dealing with the conventional policy - financial stability tradeoff[1]. The options involved have different weights in the utility of agents making hard to tell in what cases the central bank should act in an expansionary direction or not.

As hinted in several parts of the paper, some of this assumptions can limit the tractability of the model. It will be probably the case, however, it may be important to consider these modifications to exploit the explanatory potential that financial intermediaries heterogeneity dynamics (extensive and intensive margin) can have.

[1] This is a less relevant point when there is no trade-off involved since monetary policy can lead to both output expansion and financial stability.

The article is comprehensive since it explores the joint determination of the general equilibrium dynamics of the economy. It is also interesting since it links the monetary policy management with an endogenously determined systemic risk aggregate measure. Furthermore, it has the virtue of allowing for an exploration of Monetary policy that go beyond the well studied effect in real variables and is insightful in the sense that it sheds light on the non-monotonic trade-off between monetary policy conventional goals and financial macroprudential goals.

However, as mentioned before, the article model and theoretical consequenses can and should be subject to further scrutiny, in particular, a welfare accounting exercise is needed for reasons explained before.

Additionally, to mantain tractability the article has made use of a set of simplications that prevent the model from being general enough. That is reflected in the simplistic set of shocks that can be explored in the current document (only productivity and monetary). The following are the assumptions that can be improved to make the model more general:

- *Equity:* The model assumes all banks have the same level of equity as a simplification. In the same way that the VaR constraint allows to add heterogeneity, the same equity assumption takes out a heterogenity source (the size of the bank) that can be important in driving the results.

- *Monetary policy rule:* The rule assumed is very simplistic, the central bank plays no role since the model has no inflation or other kind of trade-offs. The only role of the bank is to provide insurance to the depositors. A richer structure of the bank could be included. For example, it could be interesting to see if the tradeoff between monetary policy and financial stability are present in one or both the output stabilization goals or the inflation targeting purposes.

- *Inelastic labor*: The framework abstracts completely from the labor/leisure decisions of the agents, i.e., a feature of the model is that the households only care about consumption intertemporal smoothing. Including the leisure utility can enrich the dynamics of the model and capital accumulation, making less clear the sharp decline in the returns of capital that price out safe banks and drive the main result of the paper.

- *Short lives of banks:* Finally, the assumption of two period lived banks is less than realistic. The literature finds assuming finite lives of banks as a usual practice to prevent ponzi schemes. However, a more realistic scenario is to allow the banks to leave with a given probability, such that the expected duration is finite.

In any case, it should be mentioned that the article is a great step in the direction of linking macroeconomic dynamics and financial regulations with the interplay in the heterogenous risk profile across banks. By itself the article is a great contribution, worth of further development and exploration.

Appendix

Paper description, mathematical derivations and results:

Model:

- Representative Risk Averse Household
- Continuum of Risk Neutral Financial Intermediaries
- Central Bank and Government (guarantee deposits and run a balanced budget)
- **Shocks:** Productivity and Monetary Policy (both aggregate)

However, the heterogenity at the bank level will imply that despite having aggregate shocks only, there will be idiosyncratic risks of default.

HH: Infinitely lived, they get a wage and decide to consume or save by using a storage technology or as deposits, yielding a return r_t^D. They do not invest in capital stock directly.

HH problem:

$$\max_{\{C_t,S_\tau,D_\tau^H\}} E_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$$

$$\text{s.t.} \qquad C_t + S_t^H + D_t^H = R_t D_{t-1}^H + S_{t-1}^H + W_t - T_t$$

Production techology: is given by a Cobb Douglas aggregator, since there is no disutility of working, labor is inelastically supplied, and therefore, is normalized to 1 for simplicity:

$$Y = Z_t K_{t-1}^{\theta}$$

with $\log Z_t = \rho^z \log Z_{t-1} + \varepsilon_t^z$, $\varepsilon_t^z \sim N(0, \sigma_z^2)$.
The optimality conditions of the firm imply,

$$W_t = (1-\theta) Z_t K_{t-1}^{\theta-1}$$
$$R_t^K = \theta Z_t K_{t-1}^{\theta-1} + (1-\delta)$$

Derivation of this result:

The profits of the firm will be given by:

$$\pi = f(k) - (r + \delta)k - w$$

notice that the firms discount the depreciation of capital of their profits and that labor is already set to 1 (the homogeneity of degree and the inelastic labor supply allow these changes).

First order condition is:

$$f'(k) = r + \delta$$

therefore the interest rate is $r = f'(k) - \delta$. Consider gross rates by summing one to each side, then the result will be,

$$\mathrm{R} = f'(k) + (1 - \delta)$$

as above (where for $f'(k)$ it is used the corresponding derivative of the Cobb-Douglas production function with respect to k).

To obtain the wage, consider only the marginal product of labor in the Cobb-Douglas or plug $f'(k)$ in the profit function. The result this process yields is that $w = f - f'\dot{k}$ which after replacing the CD technology yields the wage equation above.

Financial Intermediaries: they use external funding $\mathrm{D}_t^{\mathrm{H}}$ and equity ω_t to invest in capital and storage.

They live for two periods: receive and manage equity in the first period and consume their net worth in the second.

They are subject to **limited liability**: then they have higher incentives to take on risk, or higher expected return.

However, they are subject to an idiosyncratic Value at Risk (VaR) constraint that limits their leverage. The constraint governs the maximum tolerated probability of default.

Their cash flow is given by the return they get on their capital stock and the resources they stored, net of their debt service to depositors:

$$\pi_{t+1,i} = \mathrm{R}^{\mathrm{K}}_{t+1} k_{it} + s_i t - \mathrm{R}^{\mathrm{D}}_t d_{it}$$

The VaR constraint will be given by the probability of having a negative Return on Equity, that would lead to a default:

$$\mathrm{P}r(\pi_{t+1,i} < \omega) \leq \alpha_i$$

The heterogeneity in the VaR reflects different risk attitudes across banks, which in the spirit of the Basel III accord, is consistent with different implementation of the regulation requirements or even different methods for risk evaluation.

For simplicity the equity is assumed constant ($\omega = \omega^i_t$), also, the net worth consumed by the intermediary is c_{it}, clearly, when the intermediary has a negative cash flow it is zero, meaning that everything will be used to pay depositors and the government will pay the difference.

The intermediaries will be also price takers (competitive), and therefore they take r^{D}_t and the distribution of $\mathrm{R}^k_{t+1}(\varepsilon)$ as given when picking their capital stock, resources to store and funding.

The decision process of the intermediary is the following:

1. They decide whether to parcitipate or not in the market for risky assets. If they don't, the equity is fully stored.

2. Upon participating, they decide to use deposits to **lever up** (Risky Intermediary) or only invest their equility (Safe Intermediary).

Therefore, the decision to lever up determines the extensive margin of the intermediaries, whereas the amount by which they do it is related to the intensive margin. Jointly it determines the aggregate leverage and risk for the economy.
The problem of the intermediary is,

$$\mathrm{V}_{it} = \max_{\{k_{it}, s_{it}, d_{it}\}} \mathrm{E}_t(c_{i,t+1})$$

s.t.

$$\begin{aligned} Pr(\pi_{t+1,i} < \omega_t^i) &\leq \alpha^i && \text{(VaR constraint)} \\ k_{it} + s_{it} &= \omega_t^i + d_{it} && \text{(Budget constrait)} \\ c_{i,t+1} &= \max(0, \pi_{t+1,i}) \\ \pi_{t+1,i} &= R_{t+1}^{K} k_{it} + s_{it} - R_t^{D} d_{it} \end{aligned}$$

with no participation, the value of the intermedite (outside option) is just $\omega = V^O$.
In addidion there is Limited Liability which truncates the profit function at zero and then generates an option value of default. As consequence:

$$E_t[\max(0, \pi_{t+1,i})] \geq E_t[\pi_{t+1,i}]$$

The total intermediary consumption is:

$$C_t^I = \int c_{it} dG(\alpha^i)$$

i.e., we aggregate over the heterogeneity source (across banks) that follows a distribution G.
The government only funtion is to cover the deposits that the banks default, and therefore the aggregate taxes are:

$$T_t = \int t_t^i dG(\alpha^i)$$

where $t_t^i = \max(0, -\pi_{t+1,i})$
The decision is taken by comparing their value function upon entering versus the outside option value of storing their equity.

For convenience, it's assumed that when indifferent, the intermediaries will not invest any part of their resources, i.e., they will participate if $E[R_{t+1}^K] \geq 1$, more importantly, it can be proven that:

$$\text{if} \quad E[R_{t+1}^K] \geq 1 \Rightarrow d_{it} = \begin{cases} 0 & \text{no lever at all} \\ \bar{d}_t^i & \text{Maximum leverage given by VaR constraint} \end{cases}$$

Then the banks upon participating find optimal to lever as much as they can or don't do it at all. Every other intermediate leverage point is suboptimal.

Reaction of Leverage Decisions to Interest Rate and Riskyness of the Bank

Departing from the VaR restriction,

$$\Pr[\pi_{t+1,i} < \omega] \leq \alpha^i$$

We can substitute the optimality conditions of the firms in the cash flow in the VaR restriction and solve for the leverage as:

$$\lambda_{it} = \frac{k_{it}}{\omega} = \frac{r_t^D}{r_t^D - \theta Z_{t+1}^e K^{\theta-1} F^{-1}(\alpha^i) + \delta} \tag{1}$$

From here we can obtain that:

$$\frac{\partial \lambda_{it}}{\partial r_t^D} < 0, \qquad \frac{\partial^2 \lambda_{it}}{\partial (r_t^D)^2} > 0, \qquad \frac{\partial^2 \lambda_{it}}{\partial r_t^D \partial \alpha^i} < 0 \tag{2}$$

which means that: (i) leverage decreases in r_t^D and more importantly that (ii) the more risk taking the Financial intermediary, the larger the increase in leverage when r_t^D decreases.

This last feature is what implies a non-linearity between leveraging and monetary policy: Riskier agents will react more to monetary policy stimulus.

Mathematical derivation: This is arguably the most important result of the paper, it implies that there is a non-linear relation between leverage and the cross section of banks and such result will imply an increasing risk-shifting of risk per bank and higher systemic risk.

For this result we needed to get an expression for the leverage. First of all, notice that since the financial intermediates are participating, the investment in storage technology (that yields no returns) is zero, $s_{it} = 0$, then, $\pi_{t+1,i} = R_{t+1}^k k_{it} - R_t^D d_{it} = R_{t+1}^k k_{it} - R_t^D (k_{it} - \omega)$, therefore:

$$\begin{aligned}
\pi_{t+1,i} &\leq \omega \\
\theta Z_{t+1} K_t^{\theta-1} k_{it} + (1-\delta) k_{it} - R_t^D (k_{it} - \omega) &\leq \omega \\
\theta e^{\varepsilon_{t+1}^z} Z_{t+1}^e K_t^{\theta-1} - \delta - r_t^D &\leq -r_t^D \frac{\omega}{k_{it}} \\
e^{\varepsilon_{t+1}^z} &\leq \frac{r_t^D + \delta - \frac{\omega}{k_{it}} r_t^D}{\theta Z_t^e K_t^{\theta-1}}
\end{aligned}$$

in this inequalities we replaced the capital terms with the technology related associated expressions and expanded the productivity shock as the product of the expected value times the shock term itself.

now consider the VaR restriction and replace the latter result:

$$
\begin{gathered}
Pr[\pi_{t+1,i} \leq \omega] \leq \alpha^i \\
Pr[e^{\varepsilon^z_{t+1}} \leq \frac{r^D_t + \delta - \frac{\omega}{k_{it}} r^D_t}{\theta Z^e_t K^{\theta-1}_t}] \leq \alpha^i
\end{gathered}
$$

then the leverage is given by (1). Where $F^{-1}(\alpha^i)$ is the inverse cdf of the technology shock $e^{\varepsilon^z_{t+1}}$ evaluated at α^i

The derivatives in (2) can be obtained directly:

$$
\begin{aligned}
\frac{\partial \lambda_{it}}{\partial r^D_t} &= \frac{-\theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta}{(r^D_t - \theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta)^2} < 0 \\
\frac{\partial^2 \lambda_{it}}{\partial (r^D_t)^2} &= \frac{-(-\theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta)(2)(r^D_t - \theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta)}{(r^D_t - \theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta)^4} > 0
\end{aligned}
$$

it will also follow that:

$$
\frac{\partial^2 \lambda_{it}}{\partial r^D_t \partial \alpha^i} = \frac{-\theta Z^e_{t+1} K^{\theta-1} F^{-1'}(\alpha^i)(\cdot)^2 - (-\theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta)(2)(r^D_t - \theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta)(-\theta Z^e_{t+1} K^{\theta-1} F^{-1'}(\alpha^i))}{(r^D_t - \theta Z^e_{t+1} K^{\theta-1} F^{-1}(\alpha^i) + \delta)^4} < 0
$$

although such result is less clear, since it assumes a known cdf $F(\cdot)$ with positive inverse.

Extensive Margin Decision (to lever or not)

Let L denote an intermediary that levers and N one that does not use any external funding. The respective value functions are:

$$
\begin{aligned}
V^L_{it} &= E^i_t[R^K_{t+1} k_{it} - R^D_t d_{it}] \\
V^N_{it} &= E^i_t[R^K_{t+1}] k^t_{it} + w - k^N_{it}
\end{aligned}
$$

for a levered intermediary it must hold that,

$$
E^i_t[k_{it} R^K_{t+1} - R^D_t d_{it}] \geq \omega E_t[R^K_{t+1}]
$$

we can substitute $d_{it} = k_{it} - \omega$,

$$E_t^i[k_{it}(R_{t+1}^K - R_t^D) + R_t^D \omega] \geq \omega E_t[R_{t+1}^K]$$

The agent for which this inequality binds will be the marginal intermediary j that will determine the cutoff for participation $\alpha^j = \alpha^L$. Therefore, every riskier agent, i.e., those with looser VaR constraints ($\alpha \geq \alpha^L$) will lever up.
Now, we can use this equation in the case it binds, with (1) whose VaR parameter will be α^L and solve for α_t^L as,

$$\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$$

This is an implicit result since it will depend of the functional form of $F(\cdot)$ but will work as long as it's invertible.

Financial Market Equilibrium

To determine the equilibrium all we need left is to consider the market clearing constraints and to carefully aggregate through all types of intermediates.

To start with, the support of intermeriary types is $[\underline{\alpha}, \bar{\alpha}]$, the classification would be done as follows:

- those very safe (or risk averse), will not participate in the capital market: $[\underline{\alpha}, \alpha^N]$
- after that, some banks will participate but not lever up their balance sheet: $[\alpha^N, \alpha^L]$
- the rest, the riskier ones, will leverage as much as possible: $[\alpha^L, \bar{\alpha}]$

On the other hand, the market clearing constraint implies that the aggregate capital stock of the economy is the sum across intermediaries:

$$K_t = \int_{\underline{\alpha}}^{\bar{\alpha}} k_{it} \, dG(\alpha^i)$$

Systemic Risk and Crisis

The framework considered allows to consider precise definition of risk and crises.

Systemic Risk will refer to the probability of a systemic crisis and is measured directly with α_t^L.

A Systemic Crisis, will denote a situation when all levered intermediaries default, i.e., are not fully able to repay the stake holders.

In that sense, the lower the cutoff α^L, the more safe intermediaries will enter (those with low probability of default) and the lower will be the probability to distress every bank, therefore the systemic risk will decrease.

The Importance of accounting for heterogeneity of risk preferences

Will all the former ingredients in place, we can analyze why it matters to consider different risk taking across agents and the interactions between the decision to participate in the markets and the amount of leverage.

As a first approximation, we can take every other factor, different to the financial markets as given and carry out a partial equilibrium analysis. In this spirit, for different levels of the interest rates (cost of external funding) the leverage across the intermediaries types is:

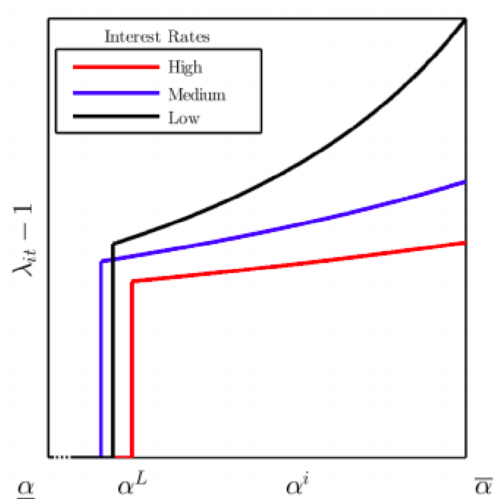

Figure 1: leverage and cross-section of financial intermediaries

For this, remember we where able to solve for the leverage for a given interest rate (that we are varying here) and VaR restriction.

The result is that: the amount of leverage (intensive margin) always increases with lower cost of funding (interest rates) but as we implied in previous results, it will increase more with lower rates.

More importantly, the area under each curve is proportional to the aggregate capital of the economy, then the higher the area, the lower the marginal return of capital. With too much leverage, and capital accumulation, the returns of capital will decrease more than what the cost of funding lowered and therefore some banks will be priced out or forced to exit the market.

That is why, when going from medium to very low interest rates, we end up with lower participation of banks or lower extensive margin.

The result is that we have a non linear effect of the interest rate in the entry/exit decisions of the banks and therefore a non-linear effect on systemic risk.

Based on that result, that is more explicit (but qualitatively equal) the main result of the paper is the following: For low values of interest rate, there is a trade-off between expanding the economy, with nometary policy, and financial stability.

By a large drop in the interest rates we obtain a way higher capital accumulation, and output. But safer firms are priced out, and excessive leverage is taken, which induces to a higher probability of default and higher systemic risk.

Further exercises of the paper, less analytical and more numerical, reveal that the very low rates induce heterogeneity in the balance sheet of the banks, the very risky banks will leverage up largely which will imply a larger skewness of leverage across the banks types.

Monetary Policy Inclusion in the model

Now, to close the model we enrich the external funding structure, for simplicity, that is done by including a monetary policy subsidy to the external funding.

The advange is within the subsidy we can include monetary policy shocks to check for the aforementioned trade-offs.

For that end we denote the central bank funding as wholesale funding with a gross interest rate of R_t^L given by,

$$R_t^L = R_t^D(1-\gamma_t)$$

then γ_t will be the central bank subsidy and will follow an AR(1) process, which allows us to link it to a shock:

$$\log \gamma_t = (1-\rho^{\gamma})\mu^{\gamma} + \rho^{\gamma} \log \gamma_{t-1} + \varepsilon_t^{\gamma} \quad \text{with } \varepsilon_t^{\gamma} \sim \mathrm{N}(0, \sigma_{\gamma})$$

here μ^{γ} will be the central bank targeted subsidy.

Nonetheless, given the subsidy, we need to assume that it will only cover a given fraction of the external funding. If it would be unlimited there would be no need for banks to use the deposits of the households.

let l_{it} be the subsidized funding (or wholesale funding) of the i-th intermediary. The wholesale funding will be assumed to cover a fixed proportion χ of the intermediary's liabilities, i.e.,

$$l_{it} = \chi d_{it}$$

This simplification allows to find the total cost of funding as:

$$[1 + \chi(1-\gamma_t)]\mathrm{R}_t^{\mathrm{D}}$$

where a unit of funding is,

$$f_{it} = (1+\chi) d_{it}$$

Making the total cost of a unit of funding be,

$$\mathrm{R}_t^{\mathrm{F}} = \frac{1 + \chi(1-\gamma_t)}{1+\gamma} \mathrm{R}_t^{\mathrm{D}}$$

Change in the balance sheet of the banks: The model has the exact structure as explained so far, however, we change the deposits for what we are denoting as the unit of external funding f, i.e., change d_{it} by f_{it} and $\mathrm{R}_t^{\mathrm{D}}$ by $\mathrm{R}_t^{\mathrm{F}}$.

Notice that with this change, we allow the central bank to change $\mathrm{R}_t^{\mathrm{F}}$ by moving γ_t (as long as there are no offsetting change in $\mathrm{R}_t^{\mathrm{D}}$)

Solution of the Model:

We look for $\mathrm{K} = \mathrm{K}^*(\mathrm{R}^{\mathrm{F}}, \mathrm{Z}^e)$ and $\alpha^{\mathrm{L}} = \alpha^{\mathrm{L}*}(\mathrm{R}^{\mathrm{F}}, \mathrm{Z}^e)$

Total funds F and deposits (supply) are:

$$F_t = \int_{\alpha^L}^{\alpha} K_{it}^L dG(\alpha^i) - [1 - G(\alpha_t^L)]\, w$$
$$D_t = \int_{\alpha^L}^{\alpha} d_{it}^L dG(\alpha^i) = \frac{F_t}{1+\chi}, with F_t = \int f_{it} dG(\alpha^i)$$

and we also have to consider the market clearing for the Deposit Market,

$$D_t^H = D_t$$

As well as the dynamics of Investment,

$$K_t = (1-\delta)K_{t-1} + I_t$$

Finally, the global economy aggregate feasibility constraint is,

$$S_{t-1}^H + S_{t-1}^I + Y_t = C_t^H + C_t^I + S_t^H + S_t^I + I_t + T_t$$

Equilibrium:

States (and shocks): $S = \{D_{t-1}, S_{t-1}^H, S_{t-1}^I, K_{t-1}, Z_{t-1}, \gamma_{t-1}, \varepsilon_t^Z, \varepsilon_t^\Upsilon\}_{t=0}^{\infty}$

The equilibrium will be given by a sequence of rates $r_{t\ t=0}^{D\infty}$ and policy rules $C_H(S), D_H(S), S_H(S), S_I(S), K(S), \alpha_L(S)$ such that the rules are optimal given $r_{t\,t=0}^{\infty}$

The mechanics of the solution method, although numeric can be synthetized as follows: for a given guess for r_t^D, impose the financial optimality equilibrium results, then fin the law of motion of wealth and consumption. After that, use the Euler Equations errors of each agent to update the deposit rate in the direction implied by the mismatch. Continue this process until the wedge is small and there is convergence.

Calibration: $A : G(\alpha^i)$ is uniform between $[0,\alpha]$, $u(c)$ is a CRRA and literature referenced as well as data estimated values are assigned to the parameters.

The solution of what remains of the paper is numerical and the main theoretical results have been already explained (the non-linear relationship between interest rate and leverage).

However, we can briefly explore one of the two shocks implemented, the monetary policy one:

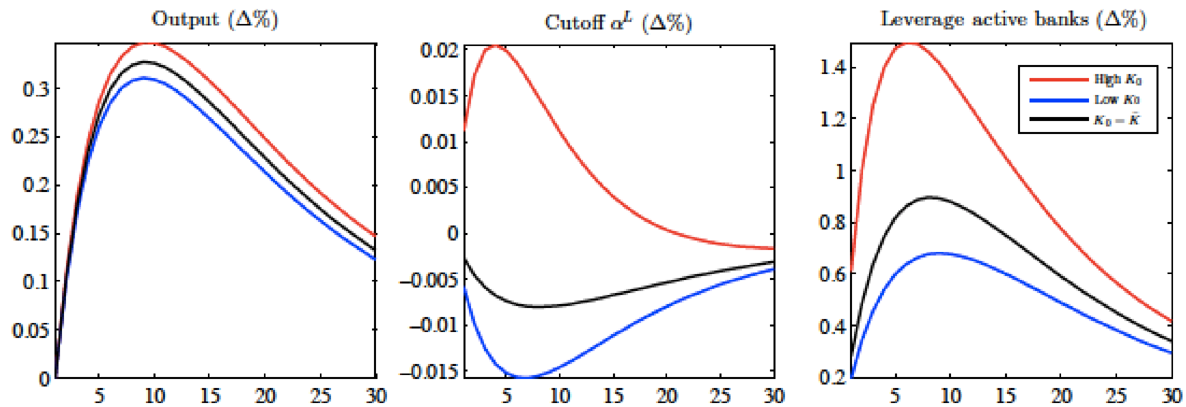

Figure 4: *Monetary policy shock of 100 basis points to* γ_t

A way of analyzing the scale dependent effect of monetary policy, that is, its effect when there are different states of the interest rate, we can vary the initial level of capital, the figure depicts the same shocks with high interest rate (blue), a medium one (black) and a low interest rate (red). Here we focus on the main variables.

The shock is that of a positive subsidy in the lending rates, i.e., a lower effective interest rate or a decrease in the cost of external funding for financial intermediaries.

It can be seen that the effect on the output is the expected, the lower the interest rate, the output and the effect of a negative shock in the interest rate is positive for all periods.

More interesting is the effect on the cutoff for leveraging. Here a lower α^L means more banks entering the market for capital funding. When we go from a high interest rate to a medium one the effect is the expected one, a decrese in the cost of funding will decrease the participation cutoff, i.e., more banks will enter the market and use external funding.

The interest case is the red one, with low interest rates, the capital is already too high and therefore the decrease in the marginal product of capital (that is proportional to the return of capital the intermediaries get) will decrease in a way that makes irrelevant the cheaper cost of funding induced by the shock. As result, the participation cutoff will increase, i.e., there will be exiting banks.

Furthermore, the banks exiting are those with a lower VaR constraint (α^i) that are those that intake less debt in the form of leverage. That explains why the leverage of the remaining (riskier) banks increases in a disproportionate way in the right panel in the figure. Overall, it implies a way higher systemic risk that came as the policy sacrifice of inducing a higher output.

Conclusions

This paper provides a GE framework for modelling a financial sector with heterogenous banks.

Heterogeneity is modelled with different VaR constraints, that with limited liability will induce different risk attitudes by banks that decide by how much to lever their balance sheets.

The framework is relevant because it is compatible with bail outs that make more appealing to take on riskier positions given the higher expected return after the government partially insures the defaulting states.

The main analytical contribution of the paper is the analysis of the interaction between the extensive margin (how many banks participate in the market of capital) and the intensive margin of the banks (by how much the lever their balance sheets). This effect, not accounted previously by the literature, generates a non-linear relationship between the interest rate shocks and the financial stability for some levels of the interest rates.

The policy trade off between expanding the economy and increasing the systemic risk is only present for low levels of interest rates where the low expected return of capital makes irrelevant to have lower costs of external funding.

In particular, the detrimental effect comes by pricing out of the market the intermediaries that take safer positions while leaving the remaining ones with even higher incentives to get more indebted.

Several extensions of the model are left for future research, by one hand, there are various simplistic assumptions that the model applies to keep tractability, these may subtract realism. On the other hand, there are many important dimensions that are worth considering when determining optimal monetary policy stances. For example a welfare analysis of the model is imperative and other important features as nominal rigidities, utility of leisure or considering the open economy case are abstracted in this first attempt to model heterogenous banks.