
ECON 6356
International Finance and Macroeconomics

Lecture 4 (part 2): The Terms of Trade, Importable Goods and Exportable Goods.

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slides

chapter 7

Importable Goods,
Exportable Goods, and
the Terms of Trade

These slides are an adjusted version of the materials for Chapter 7 of the OEM book provided by the authors

Motivation

- Thus far, we have studied economies with a single good. The same good is produced, consumed, sometimes imported, and sometimes exported.
- In reality, economies produce, consume, and trade different goods. For example, copper represents a significant fraction of Chile's exports and GDP, but a small fraction of its consumption and imports.

Three Categories of Goods Based on International Tradability

- An *importable* good is either an imported good or a good that is produced domestically but is highly substitutable with a good that can be imported.
- An *exportable* good is either an exported good or a good that is sold domestically but is highly substitutable with a good that can be exported.
- A *nontradable* good is a good that is neither imported nor exported. Nontradables are exclusively produced and consumed domestically.

Two Important Relative Prices

The three categories of goods defined in the previous slide give rise to two important relative prices:

- The *Terms of Trade* is the relative price of exportable goods in terms of importable goods.
- The *Real Exchange Rate* is the relative price of a domestic basket of consumption goods in terms of foreign baskets of consumption goods.

These relative prices receive significant attention in Macroeconomics because they either drive or reflect aggregate and sectoral movements in absorption, production, and international trade.

This Chapter

- Studies open-economy models with importable and exportable goods.
- Analyzes empirically and theoretically the aggregate consequences of movements in the terms of trade.
- The next chapter introduces nontradable goods and studies the determinants of the real exchange rate.

The Terms of Trade

Let P_t^x be a price index of exportable goods and P_t^m a price index of importable goods for a given country in period t . Then, the terms of trade for that country in period t , denoted tot_t is defined as

$$tot_t = \frac{P_t^x}{P_t^m}.$$

When tot_t increases, we say that the country experienced an improvement of its terms of trade.

$\uparrow tot_t$: "Improvement in terms of trade"

A Simple Empirical Model of the Terms of Trade

Typical country: small player in world markets for exports and imports (goods).

It makes sense to assume that variations in Terms of trade are exogenous.

(then we say: TOT are a source of aggregate fluctuations)

Accordingly, (we assume) the terms of trade follow a univariate autoregressive process:

$$\widehat{tot}_t = \rho \widehat{tot}_{t-1} + \pi \epsilon_t^{tot}, \quad (1)$$

where $\widehat{tot}_t$ denotes the log-deviation of tot_t from trend (cyclical component of tot_t), $\epsilon_t^{tot} \sim (0, 1)$ is a white noise, $\rho \in (-1, 1)$ denotes the serial correlation of the cyclical component of tot_t , and π denotes the standard deviation of the innovation in the terms-of-trade process.

Estimation

- Obtain $\widehat{tot}_t$ by removing a quadratic trend from the log of tot_t .
- Estimate equation (1) by OLS.
- Annual data from 51 poor and emerging countries spanning 1980 to 2011.
- Include only countries with at least 30 consecutive years of data.
- Data source: World Development Indicators (WDI) from the World Bank.
(File: data_annual.xls in the book's website.)

Terms of Trade Process: Estimation Results

Country-specific estimates of the equation

$$\widehat{tot}_t = \rho \widehat{tot}_{t-1} + \pi \epsilon_t^{tot};$$

Country	ρ	π	R^2
Mean	0.50	0.10	0.30
Median	0.53	0.09	0.31
Interquartile Range	$\begin{bmatrix} 0.41 \\ 0.61 \end{bmatrix}$	$\begin{bmatrix} 0.07 \\ 0.11 \end{bmatrix}$	$\begin{bmatrix} 0.19 \\ 0.39 \end{bmatrix}$

Observations

- Terms of trade are moderately volatile: Implied unconditional standard deviation at median, $\frac{\pi}{\sqrt{1-\rho^2}}$, equal to 0.11 (or 11% of trend value).
- Terms-of-trade shocks die relatively quickly: Half life at median, $\frac{\ln(1/2)}{\ln(\rho)}$, of just one year.
- Homogeneity Across Countries: For half of the countries, ρ lies in the interval (0.41,0.61) and π in the interval (0.07, 0.11).
- Moderate Fit: Median R^2 of 0.31.

Terms-Of-Trade And The Trade Balance: Empirics

Does an improvement of the terms of trade cause an improvement or a deterioration of the trade balance? $\uparrow tot \rightarrow \uparrow \downarrow tb$?

Focus of much research in open economy macroeconomics, and does not have an unambiguous theoretical answer.

We begin by addressing it from an empirical perspective.

Initial approach: Estimation of joint process for TOT and TB:

$$\widehat{tot}_t = \rho \widehat{tot}_{t-1} + \pi \epsilon_t^1, \quad (2)$$

$$\widehat{tb}_t = \alpha_0 \widehat{tot}_t + \alpha_1 \widehat{tot}_{t-1} + \rho_2 \widehat{tb}_{t-1} + \sqrt{\sigma_{22}} \epsilon_t^2 \quad (3)$$

where ϵ_t^1 and ϵ_t^2 are mean-zero i.i.d. innovations, $\widehat{tb}_t$ is a detrended measure of the trade balance, α_0 , α_1 , ρ_2 , and σ_{22} are parameters.

(As before, we estimate this system by OLS, country by country, and equation by equation, on annual data from 51 poor and emerging countries over the period 1980 to 2011.)

Identification

- Assumption: ϵ_t^1 is a terms-of-trade shock.
- Because the $\widehat{tot}_t$ appears contemporaneously in equation (3), we have that ϵ_t^1 is orthogonal to ϵ_t^2 .
- Thus, ϵ_t^2 captures shocks other than terms-of-trade shocks that affect the trade balance contemporaneously.
- Solving equations (2) and (3) for $[\widehat{tot}_t \ \widehat{tb}_t]'$ yields the SVAR system

$$\begin{bmatrix} \widehat{tot}_t \\ \widehat{tb}_t \end{bmatrix} = h_x \begin{bmatrix} \widehat{tot}_{t-1} \\ \widehat{tb}_{t-1} \end{bmatrix} + \Pi \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix}, \quad (4)$$

where

$$h_x \equiv \begin{bmatrix} \rho & 0 \\ \alpha_0 \rho + \alpha_1 & \rho_2 \end{bmatrix} \quad \text{and} \quad \Pi \equiv \begin{bmatrix} \pi & 0 \\ \alpha_0 \pi & \sqrt{\sigma_{22}} \end{bmatrix}.$$

Does the Trade Balance Change in Response to an Increase in the Terms of Trade?

The cross-country mean of the estimated TOT-TB SVAR system is

$$\begin{bmatrix} \widehat{tot}_t \\ \widehat{tb}_t \end{bmatrix} = \begin{bmatrix} 0.50 & 0 \\ -0.02 & 0.57 \end{bmatrix} \begin{bmatrix} \widehat{tot}_{t-1} \\ \widehat{tb}_{t-1} \end{bmatrix} + \begin{bmatrix} 0.10 & 0 \\ 0.008 & 0.032 \end{bmatrix} \begin{bmatrix} \epsilon_t^{tot} \\ \epsilon_t^{tb} \end{bmatrix}.$$

The element (2, 1) of Π is positive. Then, a positive TOT shock causes an improvement of the TB ($tb \uparrow$)

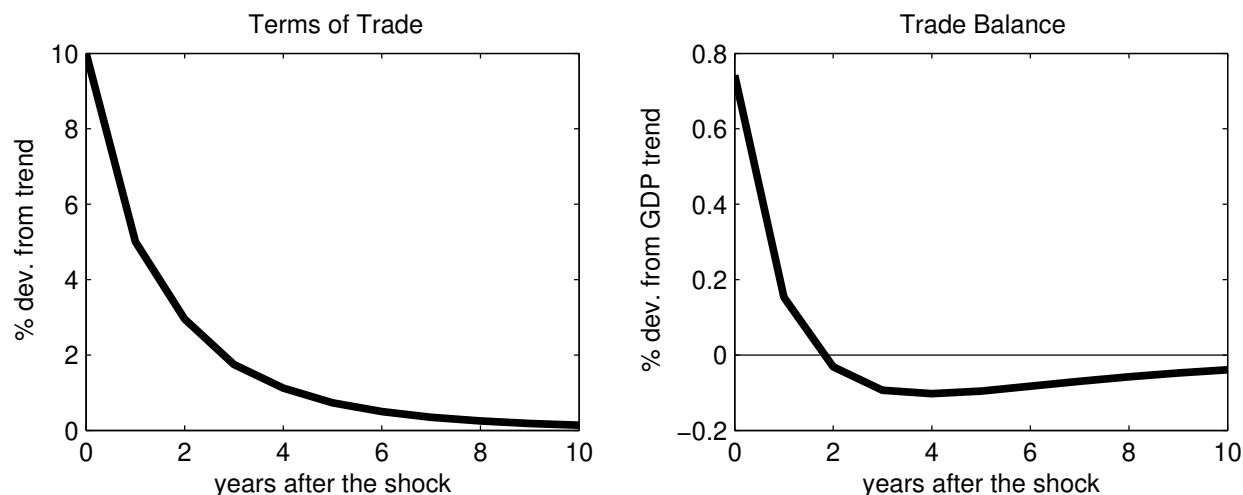
In particular, a 10 percent improvement in the terms of trade causes an improvement in the trade balance of 0.8 percent of trend GDP.

Answer: yes, the trade balance improves in response to an increase in the terms of trade.

This is true not only on average across countries, but for most individual countries (38 out of the 51 countries in the sample).

How Persistent Are the Effects of Terms-Trade-Shocks?

The figure (right panel) displays the response of the trade balance to a +10% innovation in the terms of trade (average of country-specific impulse responses).



TB improvement dies quickly: Less than two years after the improvement in the terms of trade the trade balance is below trend.

Qualification of the Previous Answer: *Yes, an increase in the terms of trade causes a short-lived improvement in the trade balance*

TOT and Trade Balance: Simple Explanations, Old and New

- **Harberger-Laursen-Metzler (HLM) Effect:** An increase in the terms of trade always improves the trade balance.

Result derived in the context of a Keynesian model and was prevalent between the early 1950s and the early 1980s.

- **Obstfeld-Svensson-Razin (ORS) Effect:** The effect of the TOT on the TB depends crucially on the persistence of the TOT.

The HLM effect holds for low persistence process, but may be reverted at high levels of persistence.

Result derived in the context of a optimizing model of the open economy.

Let's examine these two theories more closely.

The Harberger-Laursen-Metzler Effect

A classical Keynesian model delivers a positive relationship between tb_t and tot_t . Start with the national accounting identity

$$y_t = c_t + g_t + i_t + x_t - m_t,$$

where y_t = output, c_t = consumption, g_t gov't spending, i_t = investment, x_t = exports, and m_t = imports. All variables measured in units of imports.

Consider the following behavioral equations

$$g_t = \bar{g}; \quad i_t = \bar{i}; \quad c_t = \bar{c} + \alpha y_t; \quad m_t = \mu y_t,$$

where $\alpha, \mu \in (0, 1)$ are known as the marginal propensities to consume and import, and $\bar{c} + \bar{g} + \bar{i}$ as the autonomous component of domestic absorption.

Let $\bar{q}$ be the quantity of exports; then exports in units of import goods is given by

$$x_t = tot_t \bar{q}.$$

The trade balance is $tb_t = m_t - x_t$. Combining all expressions yields

$$tb_t = \frac{1-\alpha}{1+\mu-\alpha} tot_t \bar{q} - \frac{\mu(\bar{c}+\bar{g}+\bar{i})}{1+\mu-\alpha} \Rightarrow \frac{\partial tb_t}{\partial tot_t} = \frac{1-\alpha}{1+\mu-\alpha} \bar{q} > 0$$

This is the HLM effect.

Intuition: $tot_t \uparrow \Rightarrow x_t \uparrow \Rightarrow y_t \uparrow \Rightarrow c_t, m_t \uparrow \Rightarrow y_t \uparrow \Rightarrow c_t, m_t \dots$ but because $\alpha, \mu < 1$, at the end m_t increases by less than x_t , so $tb_t \uparrow$.

- The HLM effect appears to be supported by the data (see the figure on slide 14).

Terms-of-Trade Persistence and the HLM Effect

The HLM effect is mute about the effect of the persistence of tot_t .

Suppose that the marginal propensity to consume depends positively on the persistence of tot_t :

$$\alpha(\rho) \text{ with } \alpha'(\rho) > 0$$

This makes sense: consumption smoothing agents save much of temporary shocks and little of highly persistent shocks. Then we have that

$$\frac{\partial tb_t}{\partial tot_t} = \frac{1 - \alpha(\rho)}{1 + \mu - \alpha(\rho)} \bar{q},$$

$\Rightarrow$ sensitivity of tb_t to changes in tot_t decreases with the persistence of tot_t .

This is precisely what the ORS effect is about, in the context in which $\alpha(\rho)$ is endogenously derived in a utility-maximizing framework.

The Obstfeld-Svensson-Razin Effect

Consider the endowment open economy of chapter 2. Households consume an importable good, are endowed with 1 unit of an exportable good, and can borrow or lend at the rate $r > 0$. They

$$\max_{\{c_t, d_t\}} E_0 - \frac{1}{2} \sum_{t=0}^{\infty} \beta^t (c_t - \bar{c})^2,$$

subject to

$$c_t + (1 + r)d_{t-1} = tot_t + d_t,$$

Assume that tot_t follows the AR(1) process $tot_t = \rho tot_{t-1} + \epsilon_t^{tot}$. As we saw in chapter 2, the equilibrium trade balance is given by

$$tb_t = rd_{t-1} + \frac{1 - \rho}{1 + r - \rho} tot_t,$$

which implies that

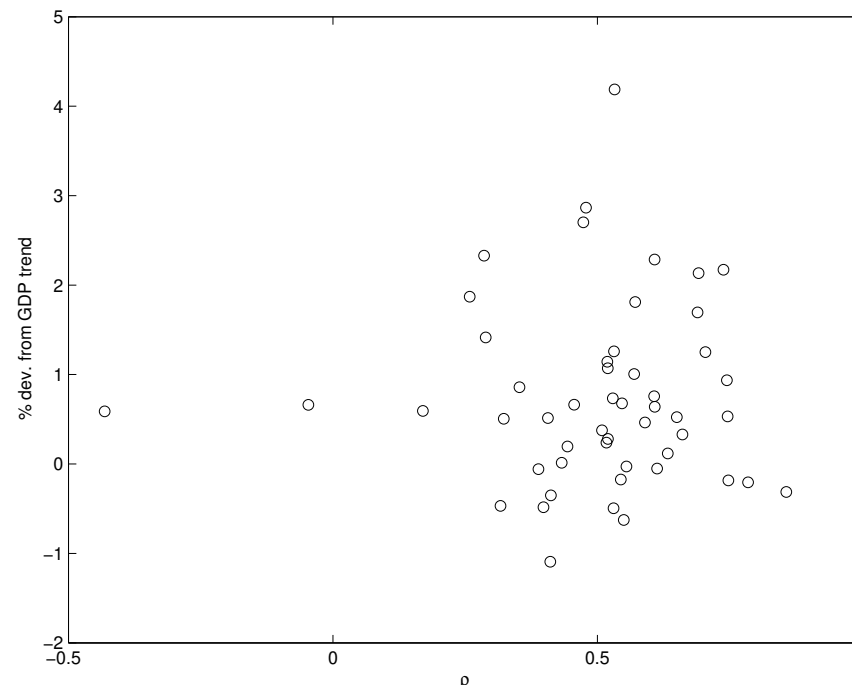
$$\frac{\partial tb_t}{\partial tot_t} = \frac{1 - \rho}{1 + r - \rho}. \quad (5)$$

The response of the trade balance to terms-of-trade shocks is weaker the more persistent the terms of trade is. This is the ORS effect.

Testing the ORS Effect

- According to the ORS effect, the response of the trade balance to terms-of-trade shocks is weaker the more persistent the terms of trade are.
- **Intuition:** a *tot* shock is like an income shock; if positive and more permanent it increases more consumption, lowering the savings (and TB) to some extent.
- Is the ORS effect borne out in the data? To see this, use the estimates of the SVAR system (4) for each of the 51 poor and emerging countries of our data set. Then, for each country plot the estimated impact response of the trade balance to a 10% increase in the terms of trade ($\frac{\pi_{21}}{\pi_{11}} \times 10$) as a function of the estimated persistence parameter ρ .
- If the ORS effect was present in the data, we should observe a negative pattern.

Testing for the ORS Effect: Impact Response of the Trade Balance to a Terms of Trade Shock and the Persistence of the Terms of Trade



Note. Each circle corresponds to a pair $(\rho, \pi_{21}/\pi_{11} \times 10)$ for a particular country, where the parameters ρ , π_{21} , and π_{11} are obtained by estimating the SVAR system (4).

The figure displays no clear negative relation between ρ and the size of the impact effect of a terms-of-trade shock on the trade balance.

Implying that the data does not lend strong support to the ORS effect.

The ORS Effect in the SOE-RBC Model

In the SOE-RBC, assume that consumption and investment are imported goods and output is exportable.

Then, assuming no productivity shocks, output in terms of importable goods is given by $tot_t F(k_t, h_t)$, and the trade balance is then given by

$$tb_t = tot_t F(k_t, h_t) - c_t - i_t - \Phi(k_{t+1} - k_t),$$

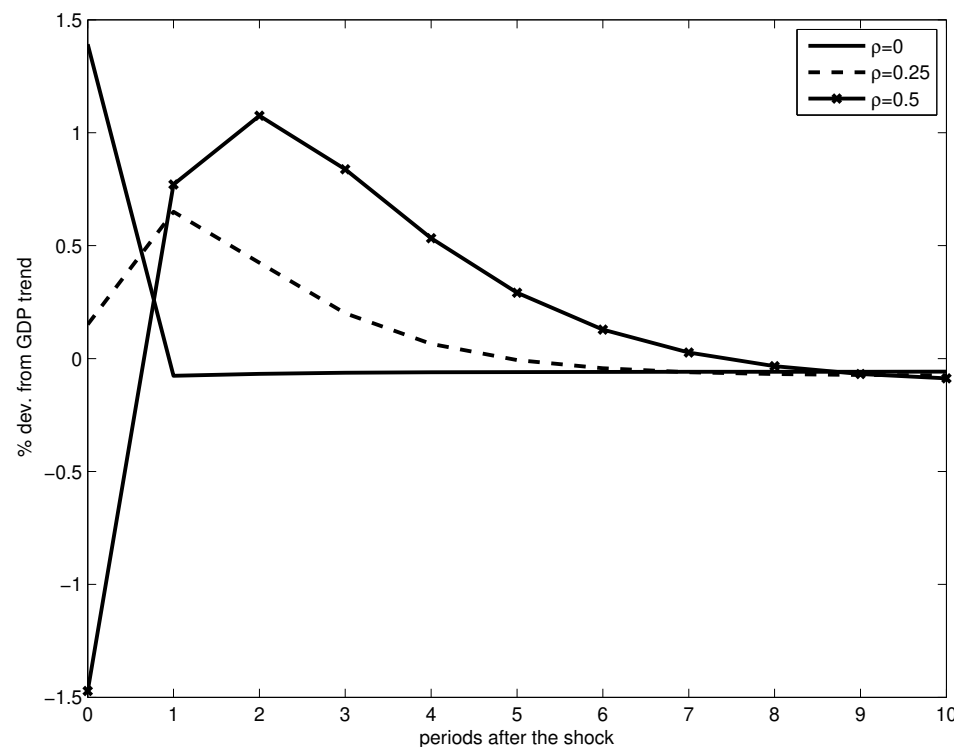
The resulting model is identical to SOE-RBC from before with A_t renamed tot_t .

Assume that the log of tot_t follows the AR(1) process $\widehat{tot}_t = \rho \widehat{tot}_{t-1} + \pi \epsilon_t^{tot}$.

In this model, $\rho \uparrow$ affects not only consumption (as in the endowment economy) but also investment. The more persistent the TOT the larger the effect of a TOT shock on investment, and the lower the effect on the trade balance.

Thus, the presence of investment strengthens the ORS effect as shown next.

Impulse Response of the Trade Balance to a Terms-of-Trade Shock Implied by the SOE-RBC Model

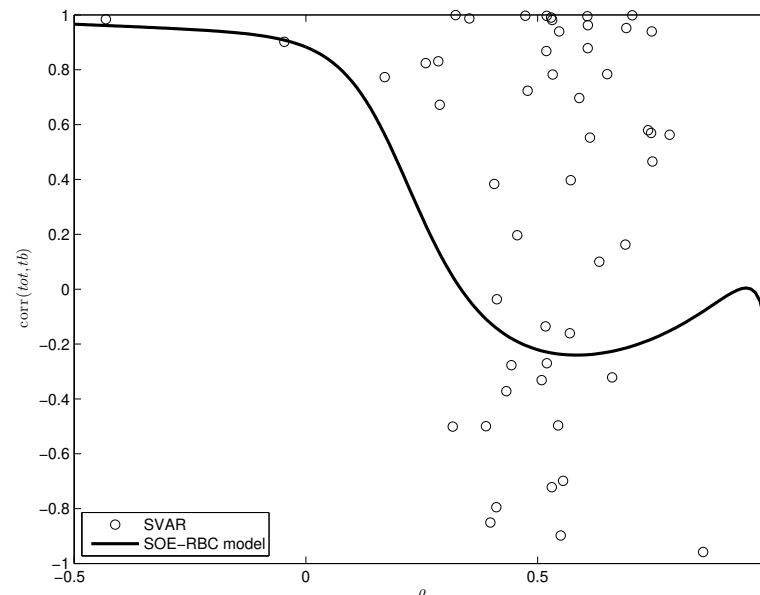


Note. Response to a one-percent increase in the terms of trade. The calibration of the model is as in chapter 4.

- The more persistent the terms of trade, the smaller the impact of a terms-of-trade shock on the trade balance.
- Thus, the ORS effect holds not only in endowment economies, but also in ones with production and capital.

Another Test of the ORS Effect

The figure displays correlations conditional on terms-of-trade shocks between the trade balance and the terms of trade implied by the SOE-RBC model (solid line) and by the empirical SVAR model given in equation (4) (circles, one for each of the 51 countries in the panel).



The SOE-RBC model is in line with the ORS effect, as it predicts a negative relation between ρ and the correlation of tb_t with tot_t conditional on tot shocks.

But no such relation is discernible in the cloud of circles. Thus, the data does not support the ORS effect from the perspective of conditional correlations.

How Important Are Terms of Trade Shocks?

Terms of trade shocks are considered important drivers of business cycles. But how important? We address this questions in two ways:

- Empirically: via a 5-variable SVAR model.
- Theoretically: the MX model.

Macroeconomic Effects of TOT shocks - Empirical Analysis

Consider the following SVAR model in five variables, the terms of trade (tot_t), the trade balance (tb_t), output (y_t), consumption (c_t), and investment (i_t):

$$x_t^1 = \rho x_{t-1}^1 + u_t^1, \quad (6)$$

$$x_t^2 = \alpha_0 x_t^1 + \alpha_1 x_{t-1}^1 + \rho_2 x_{t-1}^2 + u_t^2, \quad (7)$$

$$\text{where } x_t = \begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix}, \text{ with } x_t^1 = \widehat{tot}_t \text{ and } x_t^2 = \begin{bmatrix} \widehat{tb}_t \\ \widehat{y}_t \\ \widehat{c}_t \\ \widehat{i}_t \end{bmatrix}.$$

The scalar innovation u_t^1 and the 4-by-1 innovation u_t^2 are non-structural, i.i.d. with mean zero, and $E(u_t^1)^2 = \pi^2$ and $E u_t^2 u_t^{2'} = \Sigma$, where π is a scalar and Σ is of order 4-by-4.

Next: identification assumptions (TOT is AR(1) and other contemporaneous effects)

Identification of the Terms-of-Trade Shock

We assume that the non-structural shock $u_t \equiv [u_t^1 \ u_t^2]'$ is related to a vector of structural shocks $\epsilon_t = [\epsilon_t^1 \ \epsilon_t^2]'$ by the expression

$$\begin{bmatrix} u_t^1 \\ u_t^2 \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix}.$$

where $\epsilon \sim (\emptyset, I)$, and ϵ_t^1 and ϵ_t^2 are 1-by-1 and 4-by-1, respectively.

- We impose the restrictions $\gamma_{11} > 0$ and $\gamma_{12} = \emptyset$. This means that ϵ_t^1 is the terms of trade shock (the only innovation that affects tot_t contemporaneously).
- Because $\widehat{tot}_t$ appears as regressor in (7), we have $Eu_t^1 u_t^2 = \emptyset$, which, together with the identifying assumption of the above bullet, implies $\gamma_{21} = 0$.

Summarizing, we have

$$u_t^1 = \gamma_{11}\epsilon_t^1 \text{ and } u_t^2 = \gamma_{22}\epsilon_t^2, \text{ which implies } \gamma_{11} = \pi \text{ and } \gamma_{22}\gamma_{22}' = \Sigma$$

Rearranging terms in (6) and (7), we can then write the SVAR system as

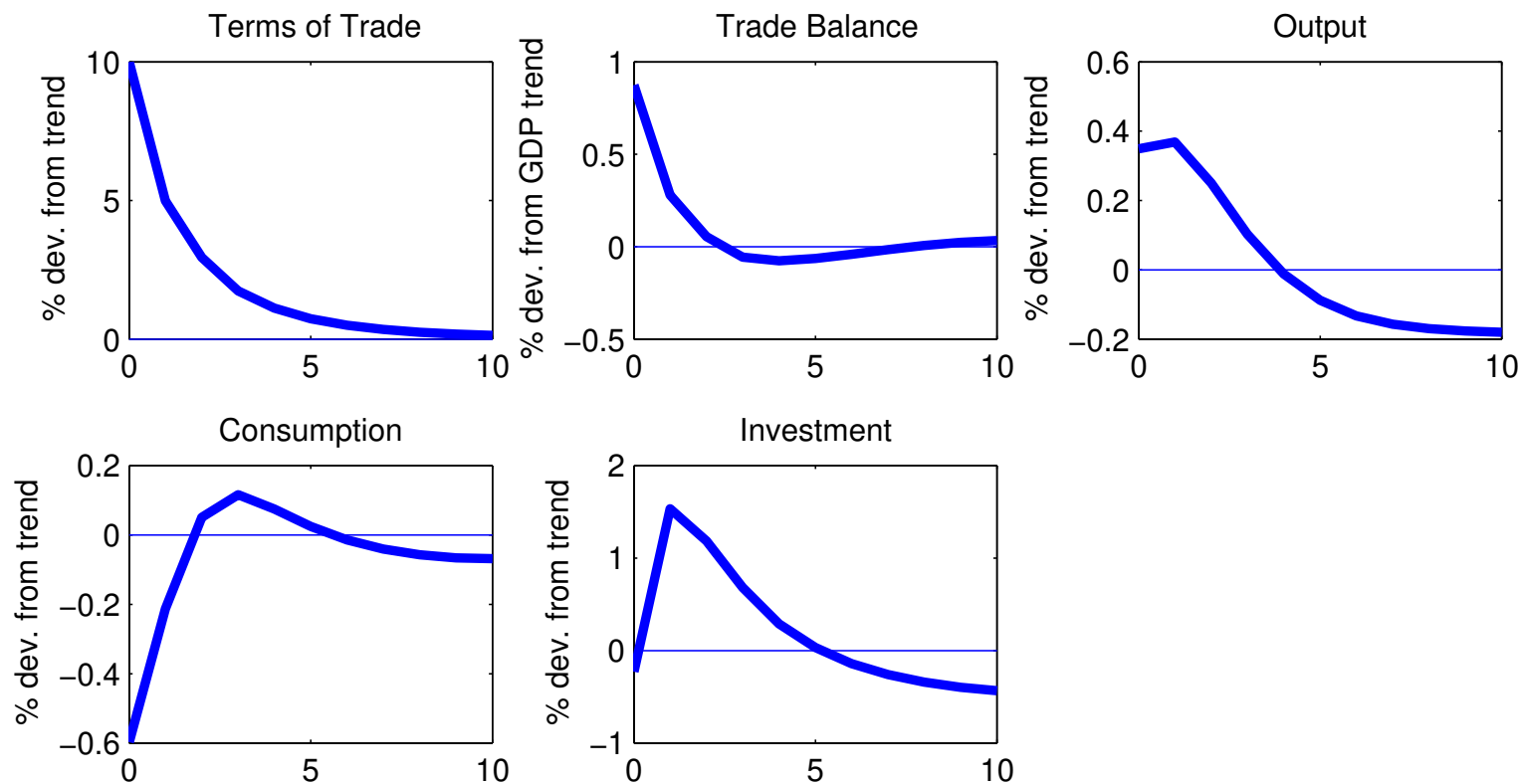
$$x_t = h_x x_{t-1} + \Pi \epsilon_t, \quad (8)$$

where $h_x \equiv \begin{bmatrix} \rho & \emptyset \\ \alpha_0 \rho + \alpha_1 & \rho_2 \end{bmatrix}$ and $\Pi \equiv \begin{bmatrix} \pi & \emptyset \\ \alpha_0 \pi & \gamma_{22} \end{bmatrix}$.

Estimation

- We estimate (6) and (7) equation by equation and country by country using the data set of 51 poor and emerging countries over the period 1980 to 2011.
- This delivers country-level estimates of ρ_1 , π , α_0 , α_1 , ρ_2 , and Σ .
- Finally, we saw that γ_{22} satisfies $\gamma_{22} \gamma'_{22} = \Sigma$. Because we are not interested in identifying the structural innovation ϵ_t^2 , any such decomposition of Σ is fine. We set γ_{22} to be the lower-triangular Cholesky decomposition of Σ .

Impulse Response to A Ten-Percent Increase in the Terms of Trade: SVAR Evidence



Note. For each variable, its impulse response is obtained by first computing the country-specific impulse responses and then averaging across countries period by period.

Comments on Impulse Responses to a Terms-of-Trade Shock

In response to a terms-of-trade shock, the following effects are observed:

- The trade balance improves. ($\Rightarrow$ The HLM effect is supported by the data.)
- Output expands for about 3 years.
- Consumption falls on impact but recovers quickly.
- Investment expands, albeit with a one-period delay.

Share of Variance Explained by Terms of Trade Shocks: Country-Level SVAR Evidence

Country	<i>tot</i>	<i>tb</i>	<i>y</i>	<i>c</i>	<i>i</i>
Cross-Country Mean	100	18	12	13	13
Cross-Country Median	100	12	10	11	10
Median Absolute Deviation	0	11	9	9	8
Using Cross-Country					
Mean of h_x and Π	100	7	3	1	2
Panel Estimation	100	4	1	1	1

Note. Variance shares are expressed in percent and are based on country-specific estimates of the SVAR system (8). Averages are taken over 51 poor and emerging countries.

Comments on Share of Variances Explained by Terms-of-Trade Shocks

- Terms of trade shocks explain a small share (10 to 12%) of the variances of output, consumption, investment, and the trade balance.
- This result is robust to estimating the variance shares in 3 ways: (1) estimating the variance shares country by country and then taking cross-country averages (baseline approach); (2) Computing an average of the country-level SVAR coefficients and using it to compute variance shares; and (3) estimating an SVAR system on a pooled panel data.
- There is significant cross-country variation in the estimated variance shares, with median absolute deviations as large as the medians themselves.

The MX Model

- Thus far, we have studied one-good models. For example, we just used the SOE-RBC model and reinterpreted the productivity shock, A_t , as a TOT shock.
- This approach is problematic: it implicitly imposes the unrealistic assumption that 100% of GDP is exported and 100% of domestic absorption is imported.
- In reality, the typical country imports only a fraction of the goods it absorbs and exports only a fraction of the goods it produces. The average trade share (the ratio of exports plus imports to GDP) is about 40 % on average.
(Under the one-good assumption, the trade share would be 200% on average across countries)
- For this reason, here we study a model economy with **two goods**, both of which are produced, consumed, and used to produce investment goods. One good is importable and the other exportable.

Households

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t^m, h_t^x),$$

subject to

$$\begin{aligned} c_t + i_t^m + i_t^x + \Phi_m(k_{t+1}^m - k_t^m) + \Phi_x(k_{t+1}^x - k_t^x) + d_t \\ = \frac{d_{t+1}}{1+r_t} + w_t^m h_t^m + w_t^x h_t^x + u_t^m k_t^m + u_t^x k_t^x, \\ k_{t+1}^m = (1 - \delta)k_t^m + i_t^m, \end{aligned} \quad (9)$$

and

$$k_{t+1}^x = (1 - \delta)k_t^x + i_t^x, \quad (10)$$

where c_t is consumption, h_t^i , i_t^j , k_t^j , w_t^j , and u_t^j hours worked, investment, capital, the real wage, and the rental rate of capital in sector j , for $j = m, x$, with m, x denoting importable and exportable sectors; d_t is debt due in period t , and r_t is the interest rate; $\Phi_i(\cdot)$ for $i = m, x$ are convex capital adjustment costs; c_t , i_t^i , w_t^i , u_t^i for $i = m, x$, and $\Phi_i(\cdot)$ are expressed in units of consumption.

Household's Optimality Conditions

- Use (9) and (10) to eliminate i_t^m and i_t^x from the sequential budget constraint.
- Let $\lambda_t \beta^t$ be the Lagrange multiplier associated with the budget constraint
- Then the FOCs with respect to c_t , h_t^m , h_t^x , d_{t+1} , k_{t+1}^m , and k_{t+1}^x are, respectively,

$$U_1(c_t, h_t^m, h_t^x) = \lambda_t, \quad (11)$$

$$-U_2(c_t, h_t^m, h_t^x) = \lambda_t w_t^m, \quad (12)$$

$$-U_3(c_t, h_t^m, h_t^x) = \lambda_t w_t^x, \quad (13)$$

$$\lambda_t = \beta(1 + r_t)E_t \lambda_{t+1}, \quad (14)$$

$$\lambda_t \left[1 + \Phi'_m(k_{t+1}^m - k_t^m) \right] = \beta E_t \lambda_{t+1} \left[u_{t+1}^m + 1 - \delta + \Phi'_m(k_{t+2}^m - k_{t+1}^m) \right], \quad (15)$$

and

$$\lambda_t \left[1 + \Phi'_x(k_{t+1}^x - k_t^x) \right] = \beta E_t \lambda_{t+1} \left[u_{t+1}^x + 1 - \delta + \Phi'_x(k_{t+2}^x - k_{t+1}^x) \right]. \quad (16)$$

Comments on the Household's Optimality Conditions

- The FOCs are similar to those of the one-sector model of Chapter 4, except that now we have sector-specific optimality conditions for capital and labor.
- In the steady state, the rental rates of capital are equalized across sectors, as adjustment costs vanish.
- Sectoral wage differentials may persist in the steady state.
- Sectoral capital adjustment cost and imperfect substitutability of sector-specific labor are intended to slow down sectoral factor reallocations
(to avoid situations such as a tractor used to produce soybeans be instantaneously flipped to the city to produce sweaters in response to an increase in the relative price of textiles.)

Production of Final Goods

Final goods are produced by firms in perfectly competitive markets. Profits are

$$A(a_t^m, a_t^x) - p_t^m a_t^m - p_t^x a_t^x,$$

where $A(a_t^m, a_t^x)$ is the quantity of final goods produced, a_t^m and a_t^x importable and exportable goods used as input, and p_t^m and p_t^x are the relative prices of importables and exportables in terms of units of the final good.

The Armington aggregator $A(a^m, a^x)$ is increasing, concave, and HD1.

Profit maximization implies that

$$A_1(a_t^m, a_t^x) = p_t^m, \tag{17}$$

$$A_2(a_t^m, a_t^x) = p_t^x. \tag{18}$$

(Armington aggregator is HD1, then firms make zero profits every period.)

Production of Importable and Exportable Goods

Importable and exportable goods are produced with capital and labor via the technologies

$$y_t^m = F^m(k_t^m, h_t^m) \quad (19)$$

$$y_t^x = F^x(k_t^x, h_t^x), \quad (20)$$

where y_t^j is output in sector $j = m, x$, and $F(\cdot, \cdot)$ is increasing, concave, and HD1. Profits are given by

$$p_t^j F^j(k_t^j, h_t^j) - w_t^j h_t^j - u_t^j k_t^j,$$

for $j = m, x$.

The optimality conditions are

$$p_t^m F_1^m(k_t^m, h_t^m) = u_t^m, \quad (21)$$

$$p_t^m F_2^m(k_t^m, h_t^m) = w_t^m, \quad (22)$$

$$p_t^x F_1^x(k_t^x, h_t^x) = u_t^x, \quad (23)$$

$$p_t^x F_2^x(k_t^x, h_t^x) = w_t^x. \quad (24)$$

Because technologies are HD1, firms make no profits.

Market Clearing

Equilibrium in the domestic final-goods market

$$c_t + i_t^m + i_t^x + \Phi_m(k_{t+1}^m - k_t^m) + \Phi_x(k_{t+1}^x - k_t^x) = A(a_t^m, a_t^x). \quad (25)$$

The equilibrium evolution of external debt

$$\frac{d_{t+1}}{1 + r_t} = d_t + m_t - x_t, \quad (26)$$

where m_t denotes imports, and x_t denotes exports (both expressed in terms of final goods) and are given by

$$m_t = p_t^m (a_t^m - y_t^m) \quad (27)$$

and

$$x_t = p_t^x (y_t^x - a_t^x). \quad (28)$$

Two Key Relative Prices

- The terms of trade (tot_t) is the relative price of exportable goods in terms of importable goods,

$$tot_t = \frac{p_t^x}{p_t^m}. \quad (29)$$

As in the SVAR model assume that the country is small and takes tot_t as exogenous. Also, assume an AR(1) structure for tot_t :

$$\ln \left(\frac{tot_t}{tot} \right) = \rho \ln \left(\frac{tot_{t-1}}{tot} \right) + \pi \epsilon_t^{tot}, \quad (30)$$

- To ensure stationarity, assume that the country interest rate premium is debt elastic as in the EDEIR model:

$$r_t - r^* = p(d_{t+1}), \quad (31)$$

where r^* denotes the world interest rate assumed to be constant. The country premium, $p(d)$, is assumed to be increasing.

The Competitive Equilibrium

A competitive equilibrium is then a set of processes $c_t, h_t^m, h_t^x, d_{t+1}, i_t^m, i_t^x, k_{t+1}^m, k_{t+1}^x, a_t^m, a_t^x, p_t^x, y_t^m, y_t^x, p_t^m, r_t, w_t^m, w_t^x, u_t^x, u_t^m, \lambda_t, m_t, x_t$, and tot_t satisfying equations (9)-(31), given initial conditions k_0^m, k_0^x, d_0 , and tot_{-1} , and the stochastic process ϵ_t^{tot} .

Observables

- How can we measure GDP in models with multiple goods?
- For a meaningful comparison of model and data, the theoretical measure of GDP must concur with its empirical counterpart.
- The WDI, from which we obtained the data for the SVAR analysis, measures real GDP as the ratio of nominal GDP to a Paasche implicit price deflator.
- The next slide explains how to obtain the appropriate theoretical counter part of the WDI's measure of real GDP.

A Data-Compatible Measure of Real GDP

Real GDP is the ratio of nominal GDP to the price level

$$\text{real GDP in } t = \frac{P_t^m y_t^m + P_t^x y_t^x}{P_t}$$

A Paasche price index in t (recall we want to comply with the WDI) is the ratio of the value of period- t quantities measured in period- t prices to the value of period- t quantities measured in base-period prices.

Denoting the base period by 0, the Paasche price index is

$$P_t = \frac{P_t^m y_t^m + P_t^x y_t^x}{P_0^m y_t^m + P_0^x y_t^x}$$

Combining the above two expressions, we obtain

$$\text{real GDP in } t = P_0^m y_t^m + P_0^x y_t^x.$$

Now since, as defined, real GDP is an *index* of real activity, we can scale its entire path by any constant without changing its meaning. Pick this constant to be the price of consumption (P_0^c) in the base year to get

$$\text{real GDP in } t = p_0^m y_t^m + p_0^x y_t^x.$$

Finally, associate the base year with the steady state to obtain

$$\text{real GDP in } t = p^m y_t^m + p^x y_t^x.$$

This measure of real GDP has two convenient properties:

- (i) The model has precise predictions for it; and
- (ii) It is as conformable to its empirical counterpart as the model allows.

(this type of variable is the one we use to compare moments with those of the data)

A Data-Compatible Measure of Other NIPA variables

All NIPA variables in the data set are deflated by the Paasche implicit price index.

Thus, for example, the appropriate theoretical counterpart of real consumption is

$$\text{real consumption in } t = \frac{P_t^c c_t}{P_t} = c_t \frac{p_t^m y_t^m + p_t^x y_t^x}{p_t^m y_t^m + p_t^x y_t^x}.$$

This expression makes it clear that comparing c_t to the measure of consumption used in the SVAR analysis would be incorrect.

Similar transformations must be applied to variables listed in the definition of a competitive equilibrium given in slide 40 to obtain the theoretical counterpart of the empirical measures of other NIPA aggregates, such as investment, imports, and exports.

Functional Forms for Preferences and Technologies

GHH preferences

$$U(c, h^m, h^x) = \frac{[c - G(h^m, h^x)]^{1-\sigma} - 1}{1-\sigma}; \quad \sigma > 0$$

Imperfect substitutability of sectoral employment

$$G(h^m, h^x) = \frac{(h^m)^{\omega_m}}{\omega_m} + \frac{(h^x)^{\omega_x}}{\omega_x}; \quad \omega_m, \omega_x > 0.$$

Cobb-Douglas technologies in the importable and exportable sectors

$$F^i(k^i, h^i) = A^i (k^i)^{\alpha_i} (h^i)^{1-\alpha_i}; \quad i = m, x; A^i > 0, \alpha_i \in (0, 1).$$

Exponential debt-elastic country premium

$$p(d) = \bar{p} + \psi (e^{d-\bar{d}} - 1); \quad \psi \geq 0.$$

Quadratic capital adjustment costs

$$\Phi_i(x) = \frac{\phi_i}{2} x^2, \quad i = m, x, \phi_i \geq 0.$$

CES Armington aggregator of importable and exportable goods,

$$A(a_t^m, a_t^x) = \left[\chi (a_t^m)^{1-\frac{1}{\mu}} + (1-\chi) (a_t^x)^{1-\frac{1}{\mu}} \right]^{\frac{1}{1-\frac{1}{\mu}}}; \quad \mu > 0, \chi \in (0, 1).$$

Parameterization of the MX Model

- Combination of calibration and estimation of the structural parameters of the MX model.
- The sources of parameter values fall into three groups:
 - ★ Parameter values common to one-good models (e.g., those of Chapters 4-6).
 - ★ Parameter values based on sectoral output and trade data.
 - ★ Parameter values estimated at the country level to match observed second moments.

Parameterization of the MX Model

Parameter Values Common to One-Sector Models

Calibrated Structural Parameters								
σ	δ	r^*	$\bar{p}$	α_m, α_x	ω_m, ω_x	tot	A^m	β
2	0.1	0.04	0.07	0.32	1.455	1	1	$(1 + r^* + \bar{p})^{-1}$

Note. The time unit is one year.

Parameterization of the MX Model

Parameter Values Based on Sectoral Output and Trade Data

μ	χ	$\bar{d}$	A^x
1	0.7399	0.0103	0.9732

Comments :

- μ : estimates at quarterly frequency $\Rightarrow \mu < 1$; estimates at 5-10 year $\Rightarrow \mu > 1$. This makes sense, as ability to substitute goods in response to price changes increases with time. The present model is calibrated at annual frequency so $\mu = 1$ seems reasonable.
- $\chi, \bar{d}, A^x$: match observed averages across time and countries of the share of the trade balance in GDP ($s_{tb} = 0.01$), the share of exports in GDP ($s_x = 0.21$), and the share of exportable output in GDP ($s_{yx} = 0.47$).

Parameterization of the MX Model

Parameter Values Estimated Country By Country to Match Observed Second Moments

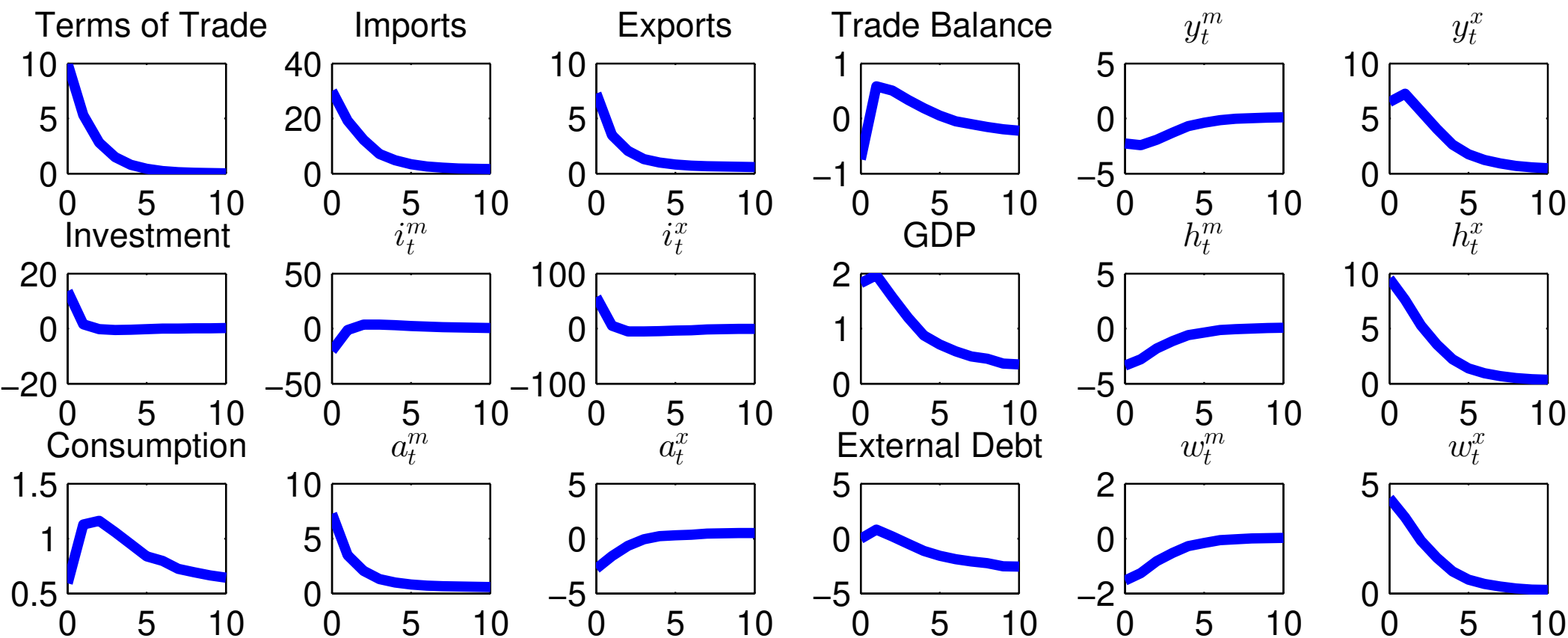
ρ	π	ϕ_m	ϕ_x	ψ
0.53	0.09	1.82	1.56	0.18

Note. Medians across 51 poor and emerging countries over the period 1980-2011.

Comments

- ρ, π : estimated using actual country-specific terms-of-trade data from the WDI.
- ϕ_m, ϕ_x, ψ : estimated by requiring the model to match the investment-output volatility ratio and the trade-balance-output volatility ratio for each of the 51 poor and emerging countries in the panel, computed over the period 1980-2011.

Impulse Responses to a Ten-Percent Improvement in the Terms-of-Trade in the MX Model



Note. All variables except for the trade balance and external debt are expressed in percent deviations from steady state. The trade balance and external debt are expressed in level deviations from steady state in percent of steady-state output. Cross-country medians.

Comments on the Response of the MX model to a Terms-of-Trade Shock

- Substitution in production away from importable goods and toward exportable goods: employment, investment, and output increase in the export sector and decline in the import sector.
- Substitution effect on domestic absorption in favor of importable goods.
- Demand for importables $\uparrow$ + Production of importables $\downarrow \Rightarrow$ imports $\uparrow$.
- Domestic absorption of exportables $\downarrow$ + Domestic production of exportables $\uparrow \Rightarrow$ Exports $\uparrow$.
- Imports and exports increase $\Rightarrow$ trade balance (? —ambiguous). In figure, TB deteriorates on impact. Thus, the model fails to capture the HLM effect.
- Appreciation of TOT $\Rightarrow$ boom in (aggr.) output, consumption, and investment. This is only partially supported by the empirical SVAR model, in which output increases, but consumption and investment fall on impact.

TOT Shocks: Less Important in Data than in Theory

How important are terms-of-trade shocks in the MX model?

And how does this prediction square with that of the empirical SVAR model?

To address these questions, the table on the next slide compares the predictions of the MX and SVAR models with respect to the shares of the variances of output and other aggregates of interest explained by terms-of-trade shocks.

Share of Variances Explained by Terms-of-Trade Shocks: The MX Model Versus the Empirical SVAR Model

	MX	Empirical
	Model	SVAR
Variable	(1)	(2)
Terms of Trade	100	100
Trade Balance	27	12
Output	18	10
Consumption	24	11
Investment	20	10

Note. Each entry is the cross-country median of the fraction of the variance of the corresponding indicator explained by terms of trade shocks in percent. In column (1) the numerator of the fraction is the variance conditional on terms-of-trade shocks predicted by the MX model for country specific calibrations of ϕ_m , ϕ_x , ψ , ρ , and π . The denominator is the unconditional variance implied by the empirical country-specific SVAR model.

Comments on the Table

- MX model: TOT shocks explain on average around 20 percent of the variances of output, consumption, investment, and the trade balance,
⇒ important source of business cycles in emerging and poor countries.
- However, the implied role assigned to terms of trade shocks by the MX model is smaller than that reported by Mendoza (1995) and Kose (2002).
- Still, the contribution of terms-of-trade shocks to business-cycle fluctuations is about twice as large in the MX model than it is in the empirical SVAR model.
⇒ Theoretical model overpredicts importance of TOT shocks.

Summary

- An SVAR estimated on data from emerging and poor countries implies that TOT shocks explain a relatively small share of the variances of macroeconomic aggregates (about 10 percent).
- A SOE-RBC with exportable and importable goods (MX model) calibrated to emerging and poor countries predicts that TOT shocks explain about 20 percent of the variances of output and other macroeconomic aggregates.
- The MX model overpredicts the importance of TOT shocks by a factor of 2.
- How can we reconcile theory and data?

Unrealistic assumption of the MX model: All goods are internationally traded. In reality 2/3 of all goods are not internationally tradable.

In the next chapter this is addressed: MX + Non-tradable goods model.